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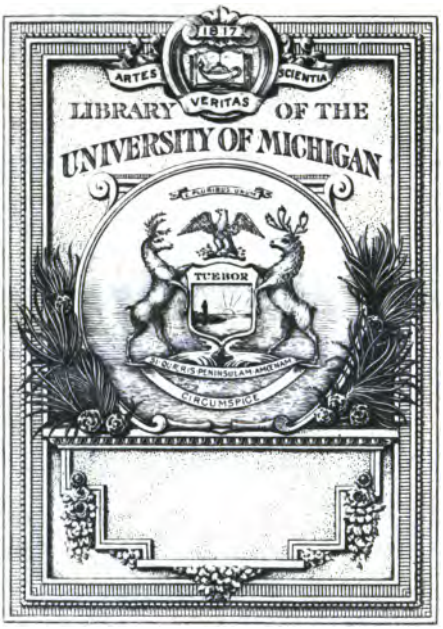

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Arithmetic

Arithmetick

THE

WESTERN ARITHMETIC;

OR

PENNSYLVANIA AND OHIO ACCOMPTANT:

BEING

A PLAIN PRACTICAL TREATISE,

WITH

A COMPLETE SYSTEM OF MENSURATION.

BY JAMES L. CONNELLY.

PITTSBURGH:
PUBLISHED BY COOK AND SCHOYER.
1835.

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CONTENTS.

	Page
ARITHMETICAL TABLES	10
Simple Addition	14
Addition of Money	16
Addition of divers denominations	17
Questions to exercise Addition	19
Simple Subtraction	21
Subtraction of Money	22
Subtraction divers denominations	23
Questions to exercise Subtraction	25
Simple Multiplication	27
Multiplication of Money	29
Multiplication of divers denominations	30
Questions to exercise Multiplication	33
Simple Division	34
Division of Money	37
Questions to exercise Division	38
Division of divers denominations	39
Reduction	41
Reduction ascending	41
Questions to exercise Reduction	42
Questions to exercise the foregoing	46
Mechanics' & Farmers' Arithmetic and Accompts	48
A Tailor's account	48
A Shoemaker's account	49
A Tanner's account	49
A Saddler's account	50
A Blacksmith's account	50
A Wheelwright's account	51
A Carpenter's account	51
A Laborer's account	51
A Farmer's account	52
Another	52
Promissory Notes, Receipts and Orders	53
The Rule of Three Direct	54
" " Inverse	58
Questions in Direct and Inverse Proportion	60
Double Rule of Three Direct	61
" " Inverse	62
Promiscuous Questions	63
Vulgar Fractions	64
Reduction of Fractions	64
Addition of Fractions	71
Subtraction of Fractions	73
Multiplication of Fractions	74
Division of Fractions	74
Rule of Three of Fractions	75
" " Inverse do.	75
Practice	76
Tare and Trett	78
Barter	81
Profit and Loss	83
Single Fellowship	87
Compound Fellowship	89
Equation of Payments	91
Single Position	92
Double Position	94
Simple Interest	99
To calculate interest on Bonds, Notes, &c.	105
Compound Interest	111
Commission	113
Insurance	114
Brokerage	115
Rebate or Discount	115
Annuities or Pensions by Simple Interest	117
Annuities or Pensions at Compound Interest	119
Permutation	123
Combination	125
Arithmetical Progression	125
Geometrical Progression	129
Alligation Medial	132
" Alternate	133
" Partial	135
Exchange	137
Decimal Fractions	139
Addition of Decimal Fractions	140
Subtraction of the same	141
Multiplication of the same	141
Division of the same	141
Contracted Multiplication of the same	143

Contracted Division of the same 144
Reduction of the same - - - 146
Square Root - - - - - - 148
Cube Root - - - - - - 152
Duodecimals - - - - - - 159
Mensuration of Superficies - 163
Of a Square - - - - - - 163
Parallelogram - - - - 163
Rhombus - - - - - 164
Rhomboides - - - - 164
Triangle - - - - - 164
Trapezium - - - - - 165
Of Irregular Figures - - - 165
Regular Polygons - - - 165
a Circle - - - - - - 166
a Semicircle - - - - - 171
a Quadrant - - - - - 172
the Sector of a Circle - - 172
the Segment of a Circle - 173
Compound Figures - - - 173
an Ellipsis - - - - - - 173
a Parabola - - - - - 174
Mensuration of Solids - - - 174
Of a Cube - - - - - - - 174
Parallelopipedon - - - 174
Triangular Prism - - 175
Pyramid - - - - - 175
Cylinder - - - - - 175
Cone - - - - - - - 175
Of the frustum of a pyramid - 176
frustum of a rectangled pyramid - - - 176
Of the frustum of a cone - - 176
Of a Sphere or Globe - - - 177
Spheroid - - - - - 178
Parabolic Conoid - - - 178
Parabolic Spindle - - 178
Artificers' Mensuration - - 179
Of Carpenters' work - - - 179
Flooring - - - - - - 179
Partitioning - - - - - 179
Roofing - - - - - - 179
Of Bricklayers' work - - - 180
Of Walling - - - - - - 180
Chimnies - - - - - - 181
Of Plasterers' work - - - 182
Rendering - - - - - 183
Of Joiners' work - - - - 183
Masons' work - - - - 183
Mensuration of boards & timber 184
Of boards - - - - - - - 184
squared timber - - - - 184
unequal squared timber - 185
Of round timber, whose bases are equal - - - - 185
Of round timber, whose bases are unequal - - - 186
Of the Five Regular Bodies - 186
Of a Tetraedron - - - - 186
Octaedron - - - - - 186
Dodecaedron - - - - 187
Icosaedron - - - - - 187
Of any irregular body - - - 188
Practical Questions - - - - 188
Winter evening Questions - 204
Appendix - - - - - - - 205
Of Pendulums - - - - - 205
the Lever - - - - - - 206
the Wheel and Axletree - 206
Water - - - - - - - 207
a Balloon - - - - - - 210
A table of specific gravities - 211
Of the specific gravities of bodies 211
How to try spirituous liquors - 212
To find the magnitude of bodies 213
Chronological Cycles - - - 214
A Perpetual Almanack - - 218
A table of the 19 Epacts - - 219
A table of the Dominical Letters 219
Useful tables - - - - - - 220
Of Gauging - - - - - - 221
To measure a Cask - - - - 221
To find the ullage of a Cask - 222
To measure a Still - - - - 222
A Key to Mensuration and the Practical Questions - - 223

PREFACE.

I SHALL not amuse the reader with any pompous eulogium on the merits of this work; or endeavour, as is customary, to anticipate the public favour by a recital of excellencies, which, in the opinion of some, may be of a disputable nature. This much I may venture to say—the adding one to the number of treatises already extant, will not appear superfluous, when we consider the importance of the subject.

The design of this work is to furnish schools with a system of practical arithmetic, in which I have, according to my knowledge in this science, arranged the different branches in a judicious and perspicuous order.

I shall not here detail the particulars in this book, but refer the reader to the table of contents.

I have entirely omitted pounds, shillings, and pence, (except a few examples in exchange with England and Ireland,) as useless to the learner; and have introduced a complete system of Mensuration, (arithmetically,) according to Euclid, and other celebrated authors.

This work is in two parts: the second part contains a useful appendix, and a key for Mensuration and the exercise questions.

Perfect accuracy, in a work of this size and nature, can hardly be expected: the author and the press are liable to errors. If any such are pointed out, it will be considered as a token of friendship and favour.

How far this treatise will answer the end intended, will, in some measure, be obvious to any judicious scholar, by taking a short view of the plan pursued.

JAMES L. CONNOLLY,
Schoolmaster.

WASHINGTON, G. C. Ohio,
May 1st, 1828.

I HAVE carefully examined Mr. Connolly's system of Arithmetic and Mensuration, and have not the least hesitation in pronouncing it to be worthy of a place among the best authors, on these subjects, in the United States.

JOHN TAYLOR, E. M.

Pittsburgh, Sept. 4th, 1828.

CONNOLLY'S ARITHMETIC.

ARITHMETIC is the art or science of computing by numbers, either whole or fractional, and is generally divided into five parts, viz. *Numeration, Addition, Subtraction, Multiplication and Division.*

NUMERATION

Is the art of expressing numbers, characters, or figures, and teaches to read and write them by their true value. Thus, one 1, two 2, three 3, four 4, five 5, six 6, seven 7, eight 8, nine 9, nought or cipher 0; and this is the reading and writing of figures.

These nine figures or digits are divided into three periods, three in each period. The first period are units, tens, and hundreds; the second period are thousands, tens of thousands, hundreds of thousands; and the third period are millions, tens of millions, hundreds of millions.

Besides the simple value of figures, each has a local value, which depends upon the place it stands in, viz. any figure in the place of units, only represents its simple value, or so many ones; but in the second place, it becomes so many tens, or ten times its simple value; and in the third place, it becomes an hundred times its simple value. The three periods of the nine digits are as follows, viz.

Hundreds of millions.	Hundreds of thousands.	Hundreds.
9 8 7	6 5 4	3 2 1

The nine digits are sufficient to express most ordinary concerns.

A cipher by itself counts nothing, but placed after a figure, makes that figure ten times greater than it was before the cipher was put to it, as 1 reads one, but 1 and a 0, (cipher,) reads ten; and a cipher put to the right hand of 10, makes that ten 100, one hundred, &c. And placed after 2 makes 20, twenty, and after 20 it makes 200, two hundred, &c. But placed before a figure, will lessen the value of that figure in the same ratio as it increased the other in decimals; but before a whole number, it neither increases nor decreases its value, unless an integer or digit be placed on the left of the cipher, as 7 is seven, or 07 is seven, and 207 is two hundred and seven.

ARITHMETICAL TABLES.

MULTIPLICATION TABLE.

2 1 is 2	3 1 is 3	4 1 is 4	5 1 is 5	6 1 is 6	7 1 is 7
2 4	2 6	2 8	2 10	2 12	2 14
3 6	3 9	3 12	3 15	3 18	3 21
4 8	4 12	4 16	4 20	4 24	4 28
5 10	5 15	5 20	5 25	5 30	5 35
6 12	6 18	6 24	6 30	6 36	6 42
7 14	7 21	7 28	7 35	7 42	7 49
8 16	8 24	8 32	8 40	8 48	8 56
9 18	9 27	9 36	9 45	9 54	9 63
10 20	10 30	10 40	10 50	10 60	10 70
11 22	11 33	11 44	11 55	11 66	11 77
12 24	12 36	12 48	12 60	12 72	12 84

8 1 is 8	9 1 is 9	10 1 is 10	11 1 is 11	12 1 is 12
2 16	2 18	2 20	2 22	2 24
3 24	3 27	3 30	3 33	3 36
4 32	4 36	4 40	4 44	4 48
5 40	5 45	5 50	5 55	5 60
6 48	6 54	6 60	6 66	6 72
7 56	7 63	7 70	7 77	7 84
8 64	8 72	8 80	8 88	8 96
9 72	9 81	9 90	9 99	9 108
10 80	10 90	10 100	10 110	10 120
11 88	11 99	11 110	11 121	11 132
12 96	12 108	12 120	12 132	12 144

To learn this table, find your multiplier in the column head, 2, 3, 4, 5, &c. and say, twice 1 is 2, twice 2 is 4, twice 3 is 6, &c. which you will see in the right-hand column.

This mode of teaching the multiplication table, is the easiest, and I presume the best, as it does not discourage the pupil to commit it to memory, as it does when in a square. It is here divided into small tasks, which appears rather a delight to the pupil, than a burden. I always adopt this plan for my pupils, and find the good effects of it.

The teacher should cause his pupils to have a perfect idea of the numeration and this table, before he allows them to begin figures.

SECTION I.

Numeration or Notation.

1 units,
2 1 tens,
3 2 1 hundreds,
4 3 2 1 thousands,
5 4 3 2 1 tens of thousands,
6 5 4 3 2 1 hundreds of thousands,
7 6 5 4 3 2 1 millions,
8 7 6 5 4 3 2 1 tens of millions,
9 8 7 6 5 4 3 2 1 hundreds of millions.

FEDERAL MONEY.

10 Mills make	1 Cent.
10 Cents	1 Dime.
10 Dimes	1 Dollar.
10 Dollars	1 Eagle.

TROY WEIGHT.

24 Grains make	1 Pennyweight.
20 Pennyweights	1 Ounce.
12 Ounces	1 Pound.

NOTE. By these denominations are weighed gold, silver and jewels.

AVOIRDUPOIS WEIGHT.

16 Drams make	1 Ounce.
16 Ounces	1 Pound.
28 Pounds	1 Quarter.
4 Quarters	1 Hundred weight.
20 Hundred weight	1 Ton.

NOTE. By this weight are weighed all coarse, heavy goods, such as iron, steel, and groceries.

APOTHECARIES' WEIGHT.

20 Grains make	1 Scruple.
3 Scruples	1 Dram.
8 Drams	1 Ounce.
12 Ounces	1 Pound.

NOTE. Apothecaries use this weight compounding medicines.

CLOTH MEASURE.

4 Nails make	1 Quarter of a yard.
4 Quarters	1 Yard.
3 Quarters	1 Ell Flemish.
5 Quarters	1 Ell English.
6 Quarters	1 Ell French.

DRY MEASURE.

2 Pints	make	1 Quart.
8 Quarts		1 Peck.
4 Pecks		1 Bushel.

NOTE. This measure is applied to grain, potatoes, apples, peaches, salt, and coals, and many other articles.

LIQUID MEASURE.

4 Gills	make	1 Pint.
2 Pints		1 Quart.
4 Quarts		1 Gallon.
31½ Gallons		1 Barrel.
42 Gallons		1 Tierce.
63 Gallons		1 Hogshead.
2 Hogsheads		1 Pipe.
2 Pipes		1 Tun.

NOTE. All liquors are measured by this measure, as also oil, vinegar, &c. 231 solid inches make a gallon.

LONG MEASURE.

3 Barley Corns	make	1 Inch.
12 Inches		1 Foot.
3 Feet		1 Yard.
5½ Yards		1 Rod.
40 Rods		1 Furlong.
8 Furlongs		1 Mile.
3 Miles		1 League.
69½ Statute or 60 Geographical Miles		1 Degree on the earth.
360 Degrees the circumference of the earth.		

NOTE. This table is used in measuring distances of places, or any thing else, where length is considered, without regard to breadth. A pole and perch is not always of the same length, for the statute measure of a pole or perch is 5½ yards. But measuring fens and woodlands, it is customary to reckon 18 feet, and for forests 21 feet to the pole or perch. In measuring the height of horses, 4 inches make 1 hand. In measuring depths at sea, or rivers, by the fathom of 6 feet. Distances of roads, &c. is measured by a chain 4 poles long, each pole 25 links, the whole chain containing 100 links.

LAND OR SQUARE MEASURE.

144 Square Inches	make	1 Square Foot.
9 Square Feet		1 Square Yard.
30¼ Square Yards		1 Square Rod.
40 Square Rods		1 Square Rood.
4 Square Roods		1 Square Acre.
640 Square Acres		1 Square Mile.

NOTE. By this the contents of any piece of land are found.

SOLID OR CUBIC MEASURE.

1728 Solid Inches	make	1 Solid Foot.
40 Solid Feet of round } 50 Solid Feet of hewn timber }		1 Ton.
128 Solid Feet, or 8 feet long, 4 feet high, and 4 feet wide }		1 Cord of wood.

NOTE. All solids, or things that have length, breadth and depth, are measured by this measure. The wine gallon contains 231 solid inches, the beer gallon 282 solid or cubic inches. A bushel contains 2150,42 solid inches.

TIME.

60 Seconds	make - - - - -	1 Minute.
60 Minutes	- - - - - - -	1 Hour.
24 Hours	- - - - - - - -	1 Day.
7 Days	- - - - - - - -	1 Week.
4 Weeks	- - - - - - - -	1 Month.
13 Months 1 day 6 hours	- -	1 Julian Year.

Thirty days hath September, April, June, and November,
February twenty-eight alone, and all the rest thirty-one.

In Leap-Year February hath 29 days, which is every fourth year.

CIRCULAR MOTION.

60 Seconds	make - - - - -	1 Minute.
60 Minutes	- - - - - - -	1 Degree.
30 Degrees	- - - - - - - -	1 Sign.

12 Signs or 360 degrees the whole great circle of the zodiac.

Explanation of Characters used in this book.

= Two parallel lines are the marks of equality; as, 100 cents = 1 dollar, or 16 ounces = 1 pound.

\+ Cross signifies more, or Addition; as, 4+6=10, that is, 4 added to 6 make the sum 10.

— Straight line signifies less, or Subtraction; as, 10—4=6, that is, 4 taken from 10 leave or equal 6.

× Denotes Multiplication; as, 4×2=8, that is, multiplied by 2 equal 8.

÷ A line between two points is the sign of Division; as, 10÷2=5, that is, 10 divided by 2 equal 5.

)(The reverse parenthesis also denote Division.

:: Four points, set in the middle of four numbers, denote them to be in Proportion to one another, by the Rule of Three; as, 4 : 6 :: 8 : 12, that is, as 4 is to 6, so is 8 to 12.

√ Prefixed to any number, supposes that the square root of that number is required.

∛ Prefixed to any number, signifies that the cube root of that number is required.

I would strongly recommend the pupil to get all the foregoing tables by heart, before he begins the several branches to which they belong. By so doing, he will receive a great benefit in his progress, and save his teacher much trouble.

By practising the numeration table in different forms, to what is here laid down, the pupil will also be enabled to read and write the following with ease.

Write down in figures, at length, the following sums:

Eleven, twenty-nine, forty-one, sixty-eight, ninety.

One hundred and one, one hundred and ninety-two, four hundred and sixty-five, nine hundred and ninety-nine.

One thousand two hundred and four, six thousand seven hundred and twenty-nine, nineteen thousand seven hundred and ninety-two, four hundred and twenty-seven thousand two hundred and ninety-six, nine hundred and forty-two thousand seven hundred.

Two millions six hundred and forty-two thousand two hundred and forty-nine, one hundred and ninety-seven millions three hundred and forty-two thousand six hundred and ninety-two.

Write down in words, at length, the following:

7, 14, 20, 29, 38, 56, 89, 102, 169, 874, 4762, 19872, 87634, 765432, 9876543, 98765432, 987654321.

SIMPLE ADDITION

Is the adding of two or more numbers together, (of the same denomination,) to make them one total sum, or number. Some authors affirm, that there are two sorts of Addition, Subtraction, Multiplication and Division, simple and compound; but I am not of that opinion. As, for instance, compound is a mixture of many particulars, of divers denominations, how can 24 dollars be added to 6 pounds of iron? or how can 20 yards of cloth be taken from 100 pounds of sugar?

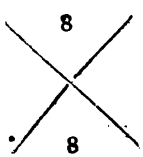
In Addition, you must place your figures in such a manner, that units will stand under units, tens under tens, hundreds under hundreds, &c. Begin at the units on the lowest line, and add upwards each column, carrying the number of tens contained in each column to the next, putting down the remainder; and the sum of the last, or left-hand column, you will put down in full.

Rules for proving Addition. After you add up all, begin at the top, and add downwards; and if that sum and the former agrees, you are right in the work. I prefer the following method; to cast the 9's out of each line, and put the excess in the upper fork of a cross, and cast the 9's out of the sum total, and put the excess in the under fork of the same cross; and if they be alike, the question is right.

654	4632	876587645321
325	6356	413212154367
979	10988	1289799799688

753274
597487
875675
516358
2742794

In casting out the 9's, begin at 8 on the bottom row, or units' place of that row, and say, 8 and 5 is 13, that is, 4 above 9, which 4 you carry to 7, (in the same row, or right-hand line of figures,) making 11, that is, 2 above 9, which 2 you carry to 4, which make 6; then carry that 6 to the next top or bottom figure of the next left-hand line, or row of figures, and proceed throughout. Thus, 6 and 7 is 13, that is, 4 above 9; that 4 and 8 make 12, that is, 3 above 9; that 3 and 7 is 10, that is, 1 above 9; that 1 and 5 is 6, and 3, (the first figure in the next row,) is 9, that is, 0, or nothing; then 6 and 4 is 10, that is, 1 above 9; that 1 and 2 is 3, and 3 in the next left-hand row is 6, and 7 is 13, that is, 4 above 9, that 0 or nothing; then 6 and 1 in the next left-hand row is 7, and 7 is 14, that is, 5 above 9; then that 5, and the 5 above the 9, (you must not count 9's in a row of figures, when proving by the cross,) is 10, that is, 1 above 9; that 1 and the next 7 in the left-hand row is 8, and 5 is 13, that is, 4 above 9; that 4 and 8 is 12, that is, 3 above 9; that 8 and 5 is 8. This 8 is set in the upper part of the cross, thus:



Then begin at the units' place of the sum total, and say, 4 and 7 is 11, that is, 2 above 9; that 2 and the next 2 is 4, and 4 is 8, and 7 is 15, that is, 6 above 9; that 6 and the last figure 2 is 8, which set in the under part of the cross; and as they are alike, the sum total is correct.

547917	97408765	91407681
691578	8714378	79172374
707249	947198	38915426
876571	75438	71456347
324756	6547	47698710
	719	98765432
	85	17546791
	9	32171917

794321	671327698	409765432
936794	719143217	982149654
549867	376097143	714067532
716985	915374691	971214716
493219	379142760	127647143
714327	794263543	715698719
140765	310754321	976541976
321987	975432794	814326714
749076	327640932	549763247

OF MONEY.

RULE. Place the numbers, according to their respective value, under each other, dollars under dollars, dimes under dimes, cents under cents, and mills under mills, and proceed exactly as in whole numbers; then place the separatrix in the sum total, directly under the separatrix points above.

EXAMPLES.

dolls. d c. m.	*dolls. cts.*
7432,7,6,4	98731,16
3147,6,1,7	19870, 7
5432,1,4,5	31968,91
16012,5,2,6	

E.	$	*d.*	*c.*	*m*	*E.*	$	*d.*	*c.*	*m.*	$	*d.*	*c.*	*m.*
69743	4	7	3	7	671	7	6	5	7	164718	7	8	7
14320	9	3	2	6	397	8	4	7	6	317102	4	1	9
81432	7	4	1	9	719	3	7	6	3	601476	8	9	4
72143	2	7	6	0	872	7	4	3	9	762143	2	1	6
31760	0	1	3	2	914	2	7	0	1	321019	7	4	3

NOTE. As accounts are kept throughout the United States in dollars and cents, and parts of a cent, instead of mills, I will give examples here in that way, for the scholar to practise. I always teach scholars under my tuition, to add and subtract the fractional parts of cents, with very little trouble. The American youths certainly can be taught the fractional parts of a cent, as easy as the English, Irish and Scotch, are taught the fractional parts of a penny.

RULE. If halves and quarters are required to be added, call a half 2, as two quarters are equal to a half of any number; and the quarters will always read one-fourth, thus, ¼, and three-quarters thus, ¾. Therefore, in adding halves and quarters, count the figure over the 4 for so many quarters, and the 2's in the halves for so many quarters; then

you have the number of quarters contained in that column, or row of halves and quarters; then find how many 4's in the whole, and carry that number to the cents, and put down the excess thus: If 2 be the excess, put down ½; if 3 be the excess, put down ¾; and if 1 be the excess, put down ¼.

EXAMPLES.

dolls.	*cts.*
472	62½
197	31½
763	18¾
871	12½
349	87½
2653	62½

Here I begin, and say, in the column of halves and quarters, 2 and 2 is 4, and 3 is 7, and 1 is 8, and 2 is 10, making 2 cents and 2 quarters, over which I put down ½, and carry the 2 cents, &c.

lbs.	*cts.*	lbs.	*cts.*	lbs.	*cts.*	lbs.	*cts.*
9437	87½	59873	12½	7163	56¼	89408	62½
7684	62½	71432	6¼	3741	18¾	78543	31¼
8943	18¾	88198	75	7194	12½	56132	18¾
5861	31¼	81976	43¾	8319	18¾	81075	37½
8329	12½	55498	93¾	1933	87½	76432	93¾
9163	43¾	78107	18¾	5198	62½	81963	43¾
5987	93¾	37619	81¼	3814	93¾		

TROY WEIGHT.

lbs.	*oz.*	*dwt.*	*gr.*	*oz.*	*dwt.*	*gr.*	*lbs.*	*oz.*	*dwt.*
24	11	14	21	11	16	17	372	9	12
19	7	10	19	9	14	21	547	6	14
26	9	16	23	10	19	16	371	11	16
19	8	12	14	8	10	14	249	10	8
37	7	10	16	7	13	15	821	7	5
73	5	18	19	9	18	17	617	5	17

AVOIRDUPOIS WEIGHT.

cwt.	*qr.*	*lbs.*	*lbs.*	*oz.*	*dr.*	*tons.*	*cwt.*	*qr.*	*lbs.*	*oz.*	*dr.*
18	2	16	26	13	14	19	14	3	21	12	14
14	3	21	19	12	15	16	12	1	16	14	12
64	1	19	32	11	10	26	17	2	18	10	9
21	2	18	24	9	8	21	15	3	27	15	14
13	3	14	16	5	13	13	18	1	19	12	10
17	1	19	18	10	11	35	16	2	12	8	11

APOTHECARIES' WEIGHT.

lbs.	oz.	dr.	sc.	gr.
3	11	7	2	19
3	9	6	1	16
9	7	4	0	12
7	4	5	2	17
8	5	3	0	10
5	8	4	1	17
3	9	6	1	15
9	8	7	0	19

lbs.	oz.	dr.	sc.	gr.
7	1	3	1	10
9	11	7	2	16
8	9	4	1	10
9	1	5	2	18
5	6	4	1	18
8	1	2	0	19
5	7	6	1	17
9	8	7	0	10

lbs.	oz.	dr.	sc.	gr.
8	8	2	1	11
9	7	1	2	16
8	5	0	1	19
5	4	6	1	17
8	7	5	2	10
7	8	3	1	18
8	9	7	2	19
9	8	6	1	10

CLOTH MEASURE.

yds.	qr.	na.
71	3	3
87	2	2
43	1	1
74	2	3
35	3	2
76	1	3
37	3	0

E.E.	qr.	na.
44	3	2
97	2	1
71	1	2
64	2	1
52	3	0
75	1	2
69	0	2

E. Fl.	qr.	na.
174	2	3
917	1	3
194	2	1
741	2	3
652	1	2
100	2	2
504	0	2

DRY MEASURE.

pk.	qt.	pt.
4	7	1
6	5	1
1	6	0
3	4	1
7	3	1
5	3	0

bu.	pk.	qt.
8	3	7
7	1	6
3	0	4
5	2	0
4	1	3
7	0	5

bu.	pk.	qt.	pt.
74	3	6	1
19	2	1	0
71	0	5	1
62	3	2	1
26	2	1	0
38	1	0	1

bu.	pk.
84	3
50	2
19	1
90	3
73	2
85	3

LIQUID MEASURE.

gal.	qt.	pt.	gill.
87	3	1	2
74	1	0	3
48	0	1	1
39	3	1	2

hhd.	gal.	qt.
76	54	3
19	37	2
91	14	1
26	38	3

tun.	hhd.	gal.	qt.
7	3	61	2
6	2	19	3
3	1	27	2
5	2	54	3

LONG MEASURE.

yds.	ft.	in.	b. c.
84	2	8	2
76	1	10	1
61	0	11	2
92	2	9	1
28	1	7	0

mils.	fur.	pole.
18	6	16
81	7	36
74	5	18
42	3	21
24	2	12

lea.	m.	fur	pa.
8	2	6	32
9	1	7	23
7	2	5	39
5	1	3	24
3	1	0	19

LAND OR SQUARE MEASURE.

A.	*R.*	*P.*
174	3	27
847	2	31
691	0	16
715	1	9
829	2	19
387	1	24

A.	*R.*	*P.*
837	2	31
106	1	17
719	0	21
643	3	26
135	2	20
387	1	19

Sq. ft.	*Sq. in.*
8	121
7	127
6	136
5	109
4	114
3	124

SOLID MEASURE.

ton.	*feet.*
74	41
69	39
83	24
62	19
39	12

cords.	*feet.*
794	124
149	119
314	86
941	37
176	116

feet.	*inches.*
641	1446
726	1563
354	1234
693	1471
713	349

TIME.

hour.	*min.*	*sec.*
34	54	40
62	36	17
81	19	36
17	52	38

day.	*hour.*	*min.*
73	21	27
64	20	53
82	19	39
19	16	26

yr.	*mo.*	*w.*	*da.*	*hour.*	*min.*
8	9	3	6	17	30
7	6	2	5	21	19
6	7	1	4	19	12
9	5	3	5	10	11

CIRCULAR MOTION.

sign.	°	′	″
9	23	45	54
8	19	16	27
7	16	24	36
6	12	17	19
5	27	19	32

sign.	°	′	″
10	16	37	24
9	21	54	45
8	20	43	19
7	23	21	32
6	27	59	59

Questions to exercise Addition.

1. A man was born in the year 1800; I demand when he will be 52 years of age? Ans. in 1852.

2. Bought 6 barrels of whiskey, each barrel containing 32 gallons; how many gallons in all? Ans. 192.

3. I have owing me 25 eagles, 12 dolls. 47 dolls. 4 dimes, 32 dollars and 50 cents; how much is the whole amount? Ans. 341 dollars 90 cents.

4. My son, James G. Connolly, was born in the year 1821, on the 28th Feb. I demand when he will be 21 years old?
Ans. 28th Feb. 1842.

5. My second son, Thomas L. Connolly, was born 21st Aug. 1823; I demand when he will be 21 years old?
Ans. 21st Aug. 1844.

6. There are two numbers, whose difference is 31, and the lesser number 62; which is the greater number?—Ans. 93.

7. I sent to Wheeling a wagon loaded with 12 bags of wheat, each bag containing 3 bushels 3 pecks; how many bushels in the whole? Ans. 45 bushels.

8. Bought 8 bars of iron, each bar weighing 28 lbs. how much is the whole weight? Ans. 2 cwt.

9. I borrowed a sum of money, and paid in part 25 dollars 62½ cents, and the remainder is 142 dollars 18¾ cents. I demand the whole sum borrowed?
Ans. 167 dollars 81¼ cents.

10. What is the sum of 4 dimes, 60 cents, 6 dimes and 40 cents? Ans. 2 dollars.

11. I sold to A 6 acres 2 roods, to B 3 acres 1 rood 20 perches, to C 8 acres 3 roods 20 perches of land; how much did I sell in all? Ans. 18 acres 3 roods.

12. Bought 18 barrels of flour from one man for 36 dolls. 31 barrels from another for 62 dolls. 87½ cts. from another 75 barrels for 196 dollars 12½ cents. How many barrels of flour did I receive, and what did they cost me?
Ans. 124 barrels, and cost 295 dollars.

13. A man took several contracts of the national turnpike road, between Washington, (Ohio,) and Zanesville, viz. the first was 62 rods, the second 87 rods, the third 197 rods, the fourth 216 rods, the fifth 317 rods. What was the whole length of his contracts? Ans. 879 rods.

14. A man had his wagon one week working on the turnpike road, for which he is to get 1 dollar 37½ cents per day. What is the week's hire? Ans. 8 dollars 25 cents.

15. Bought 4 loads of hay; the first load weighed 5 cwt. 3 qrs. 14 lbs. the second, 6 cwt. 1 qr. 18 lbs. the third, 6 cwt. 3 qrs. 27 lbs. and the fourth, 5 cwt. 1 qr. 22 lbs. What did the four loads weigh altogether?
Ans. 24 cwt. 2 qrs. 25 lbs.

16. Sold 6 silver spoons, weight and prices as follows: the first weighed 6 dwt. 14 grs. for 2 dollars 37½ cents; the second, 7 dwt. 16 grs. for 3 dolls. 12½ cts. and the other four, each 8 dwt. 10 grs. for 3 dolls. 87½ cts. apiece. How much money do the spoons come to, and what is their weight?
Ans. 21 dollars; weight 47 dwt. 22 grs.

17. If a man gets 62½ cents for one day's work, how much is his wages for four days? Ans. 2 dolls. 50 cts.

18. Paid Wm. Rabe, for 12 yds. 2 qrs. of cloth, 36 dolls. 12½ cts. for 6 yds. calico, 5 dolls. 87½ cts. for 3 yds. 3 qrs. 2 na. of velvet, 10 dolls. 31¼ cts. What quantity of cloth, calico and velvet, did I get, and how much is the amount?

Ans. 22 yds. 1 qr. 2 na.—Amount, 52 dolls. 31¼ cts.

19. Sold to sundry persons, viz. to one, 8 bushels 3 pecks of wheat, to another, 81 bushels 2 pecks corn, to another, 110 bush. oats, and to another, 39 bush. buckwheat. How many bushels did I sell in all? Ans. 239 bush. 1 pk.

20. Bought 5 pieces of linen; the first contained 52½ yds. the second, 47 yds. 2 qrs. the third, 57 yds. 3 qrs. 2 na. the fourth, 51 yds. 2 qrs. 2 na. the fifth, 50 yds. 2 qrs. How many yards were in the whole? Ans. 260 yds.

21. Bought 8 sheep: for two I paid 1 dol. 12½ cts. apiece, for three I paid 2 dolls. 18¾ cts. apiece, and for the other three I paid 2 dolls. 37½ cts. apiece. How much money did I pay in all? Ans. 15 dolls. 93¾ cts.

SUBTRACTION

Teacheth how to take a less number from a greater of the same denomination, and shew the remainder or difference, as 8 from 12, the remainder is 4.

Rule. Place the least number under the greater, units under units, tens under tens, &c. and draw a line under them. Begin at the right hand, and take each figure in the lower line from the figure above it, and set down the remainder. If the lower figure is greater than that above it, add 10 to the upper figure, and from that figure so increased take the lower, and set down the remainder, carrying 1 to the next lower figure, with which proceed as before, and so on till the whole is finished.

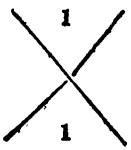
Proof. Add the remainder and lowest line together, and if the sum be equal to the upper line, the work is right: or, cast the 9's out of the upper line, and put the excess in the upper fork of a cross, and cast the 9's out of the lowest or least line, and the 9's out of the remainder. These two last excesses, added together, you will put in the under fork of the same cross; and if it be equal to the upper excess, the work is right. This last method will do in whole numbers only.

EXAMPLES.

From 876583
Take 546351

Rem. 330232

Proof.



Here the excess of 9's, in the upper line, is 1, and the excess of 9's in the lower line, and remainder, is 1, which I set down as is directed, and proves the question right.

From	647		*dolls.*		*dolls.*
Take	324	Lent	9763	Borrowed	78506
		Rec'd.	3739	Paid	32916
Rem.	323				
		Due	6024	Unpaid	45590
Proof	647				

	dolls.		*dolls.*		*dolls.*
Lent	74397	Borrowed	103876		9760
Rec'd.	39149	Paid	78907		3912
Due		Unpaid			

From	87033	67455481	307687387
Take	29147	39801976	191890519
Rem.			

From	1098763890	87602437982
Take	504002709	10009070537
Rem.		

OF MONEY.

RULE. Place the numbers under each other, according to their value, that is, dollars under dollars, dimes under dimes, cents under cents, and subtract as in whole numbers.

	dolls.	*d.*	*c.*	*m.*	*dolls.*	*d.*	*c.*	*m.*	*dolls.*	*d.*	*c.*	*m.*
From	98	7	6	5	809	8	5	6	719	7	9	8
Take	71	5	3	4	394	9	6	1	374	8	1	7
Rem.	27	2	3	1	414	8	9	5	344	9	8	1

	dolls.	*cts.*	*m.*
From	6091	75	8
Take	3914	98	2
Rem.			

	dolls.	*cts.*	*m.*
From	7439	62	8
Take	3901	1	4
Rem.			

	dolls.	*cts.*
From	9476	98
Take	1900	71
Rem.		

	dolls.	*cts.*	*m.*
From	79874	1	7
Take	50176	10	8
Rem.			

	dolls.	*cts.*	*m.*
From	8040176	19	7
Take	1900091	91	0
Rem.			

	dolls.	*cts.*
From	9767	87¼
Take	7198	31¼
Rem.	2569	56¼

Here I take one-quarter from one-half, or two-quarters, and there remains one-quarter, which I set down in its fractional form, as ¼, and proceed with the rest as in whole numbers: but if the lower fraction be more or greater than the upper one, take the lower fraction from 4, (as 4 quarters of any whole number make a whole number,) and add the remainder taken from 4 to the upper fraction, and set it down. Always be sure to carry 1 to the next whole number, when it happens that the lowest fraction is the greatest, and then proceed as in whole numbers.

	dolls.	*cts.*
From	9143	62½
Take	3917	12½
Rem.		

	dolls.	*cts.*
From	9140	12½
Take	3906	6¼
Rem.		

	dolls.	*cts.*
From	74321	18¾
Take	39714	12½
Rem.		

	dolls.	*cts.*
From	7940	6¼
Take	3596	18¾
Rem.		

	dolls.	*cts.*
From	10763	12½
Take	7190	93¾
Rem.		

	dolls.	*cts.*
From	85591	31¼
Take	59076	62½
Rem.		

TROY WEIGHT.

	lbs.	*oz.*	*dwt.*
From	6	11	14
Take	4	7	12
Rem.			

	lbs.	*oz.*	*dwt.*
From	18	10	12
Take	7	6	4
Rem.			

	lbs.	*oz.*	*dwt.*	*gr.*
From	95	6	18	16
Take	19	11	19	17
Rem.				

	lbs.	*oz.*	*dwt.*	*gr.*
From	10914	4	0	16
Take	4097	6	9	20
Rem.				

	lbs.	*oz.*	*dwt.*	*gr.*
From	9143701	1	10	3
Take	7049627	9	16	19
Rem.				

AVOIRDUPOIS WEIGHT.

	lbs.	oz.	dr.
From	7	9	8
Take	1	11	9

	cwt.	qr.	lbs.	oz.	dr.
	91	3	18	12	15
	19	2	21	13	12

	ton.	cwt.	qr.	lbs.	oz.	dr.
From	702	3	1	14	12	9
Take	391	0	3	24	13	12

	ton.	cwt.	qr.	lbs.
	9765	12	1	18
	1987	16	3	27

APOTHECARIES' WEIGHT.

	lbs.	oz.	dr.	sc.	gr.
From	974	10	2	2	19
Take	790	9	7	1	14

	lbs.	oz.	dr.	sc.	gr.
	371	9	7	2	19
	107	11	6	0	12

	lbs.	oz.	dr.	sc.	gr.
From	537	11	6	1	10
Take	191	7	0	2	19

	lbs.	oz.	dr.	sc.	gr.
	704	1	0	1	1
	298	11	7	2	19

CLOTH MEASURE.

	yds.	qr.	na.	yds.	qr.	na.	E.E	qr.	na.	E.Fl.	qr.	na.
From	9	3	2	102	1	0	76	3	2	86	1	0
Take	7	1	2	74	3	2	19	3	2	17	2	3

DRY MEASURE.

	bu.	pk.	qt.
From	75	3	7
Take	57	0	6

	bu.	pk.	qt.	pt.
	107	0	1	1
	72	3	7	0

	bu.	pk	qt.	pt.
	843	1	7	0
	190	2	6	1

LIQUID MEASURE.

	gal	qt.	pt.	gill.
From	76	3	1	0
Take	54	0	1	3

	hhd.	gal.	qt.	pt.
	87	52	3	1
	16	49	2	0

	T.	hhd.	gal.	qt.	pt.
	6	3	39	3	1
	2	1	57	3	0

LONG MEASURE.

	yds.	ft.	in.
From	37	2	11
Take	16	1	7

	mile	fur	pole.
	72	7	30
	37	0	32

	lea.	m.	fur.	po.
	8	2	7	25
	1	0	6	27

LAND MEASURE.

	A.	R.	P.
From	216	3	27
Take	179	0	37

	A.	R.	P.
From	816	0	19
Take	197	3	32

	A.	R.	P.
From	627	3	39
Take	516	3	16

	A.	R.	P.
From	762	1	16
Take	107	3	39

	A.	R.	P.
From	719	2	17
Take	206	0	30

	A.	R.	P.
From	900	1	16
Take	709	3	35

SOLID MEASURE.

	cord.	ft.
From	86	114
Take	27	120

	ton.	ft.
From	716	24
Take	382	36

	ton.	ft.	inch.
From	937	18	140
Take	591	36	127

CIRCULAR MOTION.

	sign.	°	′	″
From	49	20	51	15
Take	17	23	54	51

	sign.	°	′	″
From	172	23	45	54
Take	112	7	54	23

TIME.

	yrs.	mo.	w	da.
From	87	11	3	6
Take	53	10	2	0

	yrs.	day.	hr.	min.	sec.
From	76	352	21	41	54
Take	39	296	23	54	19

	week.	da.	hr.	min.	sec.
From	397	2	12	32	14
Take	109	5	23	54	37

	week.	da.	hr.	min.	sec.
From	796	5	21	19	36
Take	509	1	0	50	46

Questions to exercise Subtraction.

1. From fifty-four thousand and twenty-six, take nine thousand two hundred and fifty-four. Ans. 44772.

2. Subtract one from a million, and tell how many remains. Ans. 999999.

3. How much is six hundred and ten more than four hundred and five? Ans. 205.

4. What is the difference between twice twenty-five, and twice five, and twenty? Ans. 20.

5. My grandfather gave me 1000 dollars, (said Charles to John,) out of which I paid 600 dollars for a plantation, 39 dolls. 37½ cts. for building a farmhouse, 75 dolls. 87½ cts. apiece for two horses, and 12 dolls. 62½ cts. for six sheep. How much money has Charles now?

Ans. 196 dollars 25 cents.

6. From Washington, (Guernsey county, Ohio,) to Pittsburgh, is 124 miles: now, if a wagon travels four days from Washington towards Pittsburgh, at the rate of 17 miles 4 furlongs each day, how far will it be from Pittsburgh on the fourth evening? Ans. 54 miles.

7. A merchant bought 24 barrels of whiskey for 192 dolls. and sold ten barrels for 90 dolls. and one barrel he retailed for 10 dolls. 37½ cts. How many barrels has he remaining on hand, and what do they stand him?

Ans. 13 barrels—91 dolls. 62½ cts.

8. Bought a piece of cloth, containing 52 yards, for 71 dollars, and sold 19 yds. 3 qrs. 2 na. for 40 dolls. 12½ cts. How many yards have I unsold, and how much does that quantity stand me?

Ans. 32 yds. 2 na.—30 dolls. 87½ cts.

9. Thomas received from his uncle, 219 dolls. 50 cts. and lost 100 dollars 62½ cents by gambling. How much has he remaining? Ans. 118 dolls. 87½ cts.

10. Bought 2 miles 7 furlongs 37 rods of the national turnpike road to make, and sold 1 mile 5 furlongs 39 rods. How much have I yet on hand?—Ans. 1 mile 1 furlong 38 rods.

11. I received, for the rent of my plantation, of different sorts of grain, 287 bushels 3 pecks, and sold 83 bush. 2 pks. feed to my horses, 54 bush. 1 pk. and consumed in my house, 21 bush. 3 pks. How many bushels remain unsold?

Ans. 128 bushels 1 peck.

12. Received from Wheeling, 2 tons 19 cwt. 1 qr. 14 lbs. of iron, and sold 1 ton 17 cwt. 3 qrs. 27 lbs. What weight remains unsold? Ans. 1 ton 1 cwt. 1 qr. 15 lbs.

13. Bought a hogshead containing one gross* bottles of wine, and sold to Thos. Cash, threescore and seven bottles, to Richard Paywell, two dozen and four bottles, and broke in the carriage two bottles, and having company one evening, we drank eighteen bottles. How many bottles remain?

Ans. 29 bottles.

14. What sum of money taken from 900 dolls. will leave 31¼ cents behind? Ans. 899 dolls. 68¾ cts.

* A gross is 12 dozen, or 144; a score is 20; a dozen is 12, and a half dozen is 6.

15. Lodged in the bank two thousand dollars, and received, at several times, nine hundred and ninety-nine dollars ninety-nine cents. How much of my money is there yet?
Ans. 1000 dollars 1 cent.

16. War between England and America declared, June 18th, 1812, and a treaty of peace at Ghent was signed the 24th Dec. 1814. How long did the war last?
Ans. 2 years 6 months 6 days.

17. From Washington, (G. C. Ohio,) to Baltimore, is said to be 324 miles 6 furlongs, and from here to Wheeling, 39 miles 7 furlongs. How many miles from Wheeling to Baltimore? Ans. 284 miles 7 furlongs.

18. My daughter, Mary Jane Connolly, will be 18 years of age on 23d Feb. 1844. What year was she born in?
Ans. 23d Feb. 1826.

19. A silversmith bought 20 lbs. 7 oz. 9 dwt. of silver, and wrought up 12 lbs. 11 oz. 10 dwt. 12 grs. How much has he left? Ans. 7 lbs. 7 oz. 18 dwt. 12 grs.

20. A physician bought 9 lbs. 10 oz. 7 drs. 2 scr. of medicine, and has sold 5 lbs. 8 oz. 6 drs. 1 scr. What quantity has he yet on hand? Ans. 4 lbs. 2 oz. 1 dr. 1 scr.

MULTIPLICATION

Is a short way of performing the addition of several equal numbers, of the same denomination, into one sum.

The sum to be multiplied is called the multiplicand; the sum multiplied by is the multiplier; the amount produced is called the product. The multiplicand and multiplier are often called factors, and the product the rectangle.

Rule. Place the multiplicand at top, and the multiplier in the under line, in such a manner, that units will stand under units, tens under tens, &c. Then multiply them from right to left, setting down the units' figure of each particular product, and carry 1 for every 10, as in addition of whole numbers, and you will have the product, or answer.

When the multiplier consists of more figures than one, there must be made as many products as there are figures contained in the multiplier; and the first figure of every particular product must be placed exactly under its multiplier. Then add the several products, (as they stand,) together, and their sum will be the total product.

When there are ciphers at the right hand of either or both factors, omit them, until you add up the several products;

then place the ciphers, so omitted, on the right hand of the total sum, or product.

When 11 or 12 occurs, you may multiply by them at once, or separately. When any number between 12 and 20 occurs, then multiply by the units of the multiplier; and as you multiply, add to the product of each single figure, that of the multiplicand, which stands next on the right hand.

When ciphers are placed between the significant figures in the multiplier, they must be omitted in the operation, regard being had to the first figure of every particular product, as before.

When the multiplier is such a number, that any two figures multiplied together will make that multiplier, it is shorter to multiply the given number by one of those figures, and that product by the other, as 6 times 7 is 42, or 5 times 9 is 45. To multiply by 10, 100, 1000. &c. annex all the ciphers in the multiplier to the multiplicand, and it is done.

To prove Multiplication, cast the 9's out of the multiplicand, and set the excess in the left fork of a cross; then cast the 9's out of the multiplier, and set that excess in the right fork of the same cross; then multiply the two excesses together, and cast the 9's out of their product, and set the excess in the upper fork of the cross; and, lastly, cast the 9's out of the total product, and set the excess in the bottom of the cross. If the top and bottom excesses are alike, the question is right.

The pupil ought to observe the preceding directions with care: by so doing, he will reap benefit, not only of these, but what is to follow.

Multiply	432	7680	74324	987632
By	2	3	4	5
	864	23040	297296	4938160

Multiply	79832	854327	56743207
By	6	7	8

Multiply	7607543145	9876056921
By	9	10

Multiply	56843214987	904325106873
By	11	12

Multiply	75376875		90563720198
By	13		14
Multiply	763876	987530	8765102
By	15	16	17
		691861	129186
	5	26	98
		17988386	
	4 8		
	5	37451	281216
		89	76

1	Multiply	281216	by 987	Ans. 275029248
2		269181	4629	1246058849
3		261986	7638	2001049068
4		812617	43859	$5640569003
5		281691	76286	21489079626
6		476000	170	80920000
7		180120	48100	8663772000
8		461210	81900	$7773099000
9		760000	4800	$648000000
10		461200	7200	$320640000
11		618010	74210	45862522100
12		128121	72001	9224840121

OF MONEY.

RULE. When there are eagles, dollars, dimes, cents or mills, multiply as in whole numbers, and point off to the right hand, according to the quality given. If you were desired to multiply 2 dollars 50 cents by 3, say, $2,50\times3=7,50$, that is, 7 dolls. 50 cts. Here are 50 cents to the right hand of dollars, therefore you must point off two figures in the product for cents, and all to the left of the cents are dollars. If cents and mills are given, point off one figure for mills, and all the rest are cents. If dimes and cents are given, point off one figure for cents, and the figures to the left hand are dimes. If dollars and dimes are given, point off one figure as before, and all the rest are dollars. If eagles and dollars are given, point off one figure to the right hand, as before directed, in the product, and all to the left hand are eagles, and to the right dollars.

EXAMPLES.

	£	cts.	£	cts.	£	cts.	£	cts.
Multiply	6	25	9	50	32	75	98	84
By		5		8		9		12
Ans.	31	25	76	00	294	75	1186	08

		cts.			£	cts.
1	Multiply	9	by	60	Ans.	5 40
2		12½		4		50
3		6¼		8		50
4		18¾		4		75
5		31¼		8	2	50
6		37½		12	4	40
7		43¾		8	3	50
8		56¼		12	6	75
9		62½		7	4	37½
10		68¾		9	6	18¾
11		81¼		9	7	31¼
12		87½		7	6	12½
13		93¾		6	5	62½

		E.	£	d.	cts.	m.		E.	dolls.	cts.
14	Multiply	4	5	3	7	5	by 8	Ans.	363	
15			1		4		305		317	20
16					41	5	150		62	25
17			9				50		450	
18		9	2				8	73	6	
19			7		50		100		750	
20			1		6¼		4		4	25
21			4		12½		5		20	62½
22			7		18¾		6		43	12½
23			9		31¼		7		65	18¾

	dolls.	cts.		
Multiply	39	75	by	72
		9		
	357	75		
		8		
Ans.	2862	00		

NOTE. Here 9 times 8 is 72. When the multiplier is large, take two or more numbers, that, being multiplied together, will make the multiplier, and proceed as is here shewn.

TROY WEIGHT.

	lbs.	oz.	dwt.	gr.	lbs.	oz.	dwt.	gr.	lbs.	oz.	dwt.	gr.
Multiply	5	11	16	4	9	5	12	14	7	9	18	20
By				2				6				8
Ans.	11	11	12	8	56	9	15	12	62	7	10	16

	lbs.	oz.	dwt.	grs.	lbs.	oz.	dwt.	grs.
Multiply	79	9	14	19	790	11	16	23
By				9				12

AVOIRDUPOIS WEIGHT.

	cwt.	*qr.*	*lbs.*	*ton.*	*cwt.*	*qr.*	*lbs.*	*ton.*	*cwt.*	*qr.*	*lbs.*	*oz.*
Multiply	2	1	14	8	14	3	12	5	6	2	16	10
By			2				3					4
Ans.	4	3	0	26	4	2	8	21	6	2	10	8

	ton.	*cwt.*	*qr.*	*lbs.*	*oz.*	*drs.*	*ton.*	*cwt.*	*qr.*	*lbs.*	*oz.*	*dr.*
Multiply	87	19	3	27	15	14	98	17	1	19	0	4
By						14						16

APOTHECARIES' WEIGHT.

	scr.	*grs.*	*dr.*	*scr.*	*grs.*	*lbs.*	*oz.*	*dr.*	*scr.*	*grs.*
Multiply	2	14	7	2	10	4	8	6	1	12
By		2			4					6
Ans.	5	8	31	1	0	28	4	7	0	12

	lbs.	*oz.*	*dr.*	*scr.*	*grs.*	*lbs.*	*oz.*	*dr.*	*scr.*	*grs.*
Multiply	76	9	7	2	19	374	11	7	2	16
By			9×8=72					8×8=64		

CLOTH MEASURE.

	yds.	*qr.*	*yds.*	*qr.*	*na.*	*yds.*	*qr.*	*na.*
Multiply	6	2	9	3	2	34	2	3
By		4			6			8
Ans.	26	0						

	yds.	*qr.*	*na.*	*E.E.*	*qr.*	*na.*	*E.Fl.*	*qr.*	*na.*
Multiply	72	3	3	59	4	3	89	2	3
By			48			54			60

DRY MEASURE.

	bu.	*pk.*	*bu.*	*pk.*	*gal.*	*qt.*	*bu.*	*pk.*	*qt.*
Multiply	8	2	84	3	1	3	98	2	7
By		4				6			8
Ans.	34	0							

	bu.	*pk.*	*gal.*	*qt.*	*pt.*	*bu.*	*pk.*	*gal.*	*qt.*	*pt.*
Multiply	74	3	1	2	1	342	2	0	2	1
By					88					81

LIQUID MEASURE.

	hhd.	*gal.*	*qt.*	*pt.*		*hhd.*	*gal.*	*qt.*	*pt.*	*gill.*
Multiply	2	26	2	1		64	54	3	1	1
By				4						8
Ans.	9	43	2	0						

	tun.	*p.*	*hhd.*	*gal.*	*qt.*	*pt.*		*tun.*	*p.*	*hhd.*	*gal.*	*qt.*	*pt.*
Multiply	9	1	1	59	3	1		870	0	1	61	2	1
By						63							84

LONG MEASURE.

	mile.	*fur.*	*pole.*		*leag.*	*m.*	*fur.*	*pole.*	*yd.*	*ft.*	*inc.*
Multiply	8	4	20		6	2	7	32	4	2	9
By			6								55
Ans.	68	4	0								

LAND MEASURE.

	A.	*R.*	*P.*		*A.*	*R.*	*P.*		*A.*	*R.*	*P.*
Multiply	9	3	24		62	2	36		187	3	4
By			4				12				36
Ans.	39	2	16								

	A.	*R.*	*P.*	*yds.*		*A.*	*R.*	*P.*	*yds.*	*ft.*	*inc.*
Multiply	374	2	24	25		564	3	37	26	6	140
By				100							120

SOLID OR CUBIC MEASURE.

	cord.	*feet.*		*cord.*	*feet.*		*ton.*	*feet.*	
Multiply	8	120		79	64		8	40	*hewn timber.*
By		4			6			8	
Ans.	35	96							

	cord.	*feet.*	*inches.*		*ton.*	*feet.*	*inches.*	
Multiply	64	119	1640		324	39	1700	*round timber.*
By			96				108	

TIME.

	yrs.	*mo.*	*w.*		*yrs.*	*mo.*	*w.*	*da.*	*hr.*	*min.*	*sec.*
Multiply	26	8	3		76	11	3	6	21	54	37
By			4								8
Ans.	106	9	0								

NOTE. When 4 weeks make a month, then the months in a year are 13.

CIRCULAR MOTION.

	sign.	°	′	″		sign.	°	′	″
Multiply	4	20	14	12		8	29	59	54
By				4					16
Ans.	18	20	56	48					

Questions to exercise Multiplication.

1. Sold 8 dozen of eggs, at 5 cents per dozen; how much is the amount? Ans. 40 cents.

2. Sold 25 pounds of butter, at 6¼ cents per pound; how much money am I to receive? Ans. 1 dol. 56¼ cts.

3. Bought 34 pounds of sugar, for 8 cents a pound; how much am I to pay for the whole? Ans. 2 dolls. 72 cts.

4. Sold 6 calves for 2 dolls. 12½ cts. a head; how much do they come to? Ans. 12 dolls. 75 cts.

5. Sold 24 bushels of corn, for 31¼ cents per bushel; how much does the corn come to? Ans. 7 dolls. 50 cts.

6. I wrought 14 days on the turnpike road, at 62½ cts. a day; how much must I receive for my wages? Ans. 8 dolls. 75 cts.

7. Thrashed 24 days, at 87½ cents per day; how much is my wages? Ans. 21 dollars.

8. What is the amount of 75 yards of Irish linen, at 50 cents per yard? Ans. 37 dolls. 50 cts.

9. What is the amount of 4 pieces of Irish linen, containing 52, 44, 50, and 38 yards, at 56¼ cents per yard? Ans. 103 dolls. 50 cts.

10. What cost 144 pounds of tea, at 2 dollars 12½ cents per pound? Ans. 306 dollars.

11. What is the value of 120 yards of satin, at 4 dollars 6¼ cents per yard? Ans. 487 dolls. 50 cts.

12. Bought 75 head of sheep, at 1 dollar 12½ cts. apiece; how much do they come to? Ans. 84 dolls. 37½ cts.

13. What does 17 tons of hay come to, at 25 cents per hundred weight? Ans. 85 dollars.

14. What does 214 pairs of men's shoes cost, at 1 dollar 12½ cents per pair? Ans. 240 dolls. 75 cts.

15. What will 12 dozen pairs of gloves come to, at 62½ cents per pair? Ans. 90 dollars.

16. Bought 56 horses for the cavalry, at 58 dolls. 50 cts. apiece; how much is the amount? Ans. 3276 dolls.

17. What does 120 barrels of flour come to, at 5 dollars 75 cents per barrel? Ans. 690 dollars.

18. Shipped off 316 barrels of pork, at 7 dolls. 25 cts. per barrel; custom-house charges, 18¾ cents per barrel; porterage and carterage, 6¼ cents a barrel. How much does the pork stand me in all? Ans. 2370 dollars.

19. Received 72 barrels of whiskey; for 20 of them I am to pay at the rate of 6 dolls. 25 cts. apiece, for 30 of them, 7 dolls. 12½ cts. apiece, and for the remainder, 8 dollars 6¼ cents apiece. How much money do they all come to?
Ans. 516 dolls. 12½ cts.

20. Sold 24 dozen of men's hats, at 5 dollars 18¾ cents apiece; how much do they come to? Ans. 1494 dolls.

21. Thomas received of his father, a certain quantity of apples, and he gave James half of what he received, and half an apple over; to John he gave half of what remained, and half an apple over; to William he gave half of the remainder, and half an apple over; then he had but two left for himself. How many apples did his father give him at first?
Ans. 23 apples.

22. A farmer put a certain number of geese into his stubble fields, and on the second day he found one-half of them dead, and half a one over; and when he counted them again, one-half of what survived at first was dead, and half a one over; and the next day, one-half of what remained was dead, and half a one over. He then drove the remainder into his pasture field, and found them to be 28. How many had he at first? Ans. 231 geese.

DIVISION

Shews how often one number is contained in another, and what remains. It is a short way of performing the continual subtraction of several equal numbers. Four principal parts are to be noticed in Division: first, the dividend, or number given to be divided; second, the divisor, or number given to be divided by; third, the quotient, or answer to the question, which shews how often the divisor is contained in the dividend; and fourth, the remainder, which must be always less than the divisor, and of the same name with the dividend.

Rule. First place your divisor to the left hand of the dividend, and then seek how many times the divisor is contained in as many figures of the dividend as are necessary. Find the greatest figure that the divisor can be multiplied by, so as to produce a product that shall not exceed that

part of the dividend used, and when found, place the figure in the quotient. Multiply the divisor by that quotient figure, and place the product under that part of the dividend used; then subtract it therefrom, and bring down the next figure of the dividend, and place it on the right hand of the remainder found by the last subtraction. After which you must seek as before, multiply and subtract, till you have brought down all the figures in the dividend.

PROOF. Multiply the divisor and quotient together, and add in the remainder, (if there be any.) If the work be right, this product or sum will be equal to the dividend.

Proof by the cross. Cast the 9's out of the divisor, and place the excess on the left hand of the cross; do the same with the quotient, and place the excess on the right hand. Multiply these two figures together, add their product to the remainder, cast the 9's out, and place the excess at top. Cast the 9's out of the dividend, and place that excess at bottom. If the work is right, the top and bottom figures will be alike.

SHORT DIVISION

Is when the divisor does not exceed 12.

RULE. Consider how many times the divisor is contained in the first figure or figures of the dividend, put the result under, and carry as many tens to the next figure, as there are ones over. Divide every figure of the dividend in the same manner, till the whole be divided.

LONG DIVISION

Is when the divisor exceeds 12.

RULE. Try how often the divisor is contained in the same number of figures of the dividend, and proceed as is shewn in the first rule. But if you cannot put the divisor into the same number of figures of the dividend, annex another figure of the dividend, and seek how many times your divisor will go into them, and proceed as directed.

When the divisor is such a number, that any two figures multiplied together will make the said divisor, it is shorter to divide the given number by one of those figures, and that quotient by the other, as 6 times 7 is 42. Then, to find your true remainder, you must multiply the first divisor into the last remainder, and to the product add the first remainder.

When there are ciphers at the right hand of the divisor, cut off the ciphers in the divisor, and as many figures from

the right hand of the dividend; then divide the remaining ones as before directed, and to the remainder, (if any,) annex those figures cut off from the dividend, and you will have the true remainder. When the divisor is 10, 100, 1000, &c. cut off as many figures from the right hand of the dividend, as there are ciphers in the divisor; and those figures, so cut off, are the remainder, and the other figures of the dividend are the quotient.

EXAMPLES.

2)846724	3)965478	4)763204328
423362	321826	190801082
2	3	4
846724	965478	763204328

Dividend. Quotient.

Divisor 14)1785687536723(127549109765+13

14

38
28

105
98

76
70

68
56

127
126

15
14

136
126

107
98

92
84

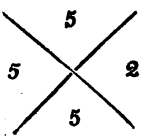
83
70

13 Rem.

14

1785687536723 Proof by Multiplication.

Proof



by the Cross.

7)10943
1563+2
7
10943

6)9014583
1502430+3
6
9014583

5)76302981
15260596+1
5
76302981

9)1476327

11)907654

12)97532741

15)1987655927

19)10976532740

1	Divide	863256	by	736	Ans. 1172
2		9870		42	235
3		94979		41	2316+23
4		1893312		912	2076
5		1893312		2076	912
6		330098148		4207	78464
7		7654321		10	765432+1
8		9876543		20	493827+3
9		1234567		30	41152+7
10		5678903		40	141972+23
11		6732761		50	134655+11
12		9103214		100	91032+14
13		8765327		1000	8765+327
14		9163000		1000	9163
15		3256700		100	32567
16		8760000		10000	876
17		9800000		9800000	1
18		76021432		6936	
19		80302543		87654	
20		7650325		10437	
21		9801432		90435	
22		1000000		76543	

OF MONEY.

RULE. If the given number to be divided consists of eagles, dollars, dimes, cents and mills, bring down the given number, and divide as in whole numbers, and the quotient will be the same name as the given sum in its reduced state, before it was divided; then point off the quotient figures in the following manner: Point off one figure to the right hand for mills, one to the left of that for cents, one to the left of that for dimes, one to the left of that for dollars, and all to

the left of dollars are eagles. If dollars and cents are to be divided, divide as in whole numbers, and point off two figures to the right for cents, and all to the left of cents are dollars. If the cents in the given sum to be divided consist of but one figure, as 2 dolls. 6 cts. be always certain you put a cipher on the left hand of the cents given; or, in other words, in the tens places of cents, as 2 dolls. 06 cts. otherwise your work will not be right.

EXAMPLES.

Divide 26 dollars by 4.	Divide 39 dollars by 5.
4)2600 cts.	5)3900 cts.
$6,50 cts.	$7,80 cts.

Here I bring down the 26 dollars, and annex two ciphers, making 2600 cents; then divide 4 into that number, and the quotient is 650 cents, in which I point off two figures to the right hand for cents, and all to the left of cents are dollars; and so proceed.

		dolls. cts. m.		*dolls. cts. m.*
1	Divide	765 50 5 by	5	Ans. 153 10 1
2		395 50	8	49 43+6=$\frac{3}{4}$
3		104	6	17 33+2=$\frac{1}{3}$
4		142	8	17 75
5		216	10	21 60
6		370	12	30 83+4=$\frac{1}{2}$
7		416	12	34 66+8=$\frac{2}{3}$

Questions for exercise.

1. Paid 20 labourers 50 dollars; what money doth each receive? Ans. 2 dolls. 50 cts.

2. Bought 18 hogs for 22 dollars 50 cents; what is that apiece? Ans. 1 dol. 25 cts.

3. Sold 24 pounds of tallow for 3 dollars; what is that a pound? Ans. 12½ cents.

4. Bought 4 dozen and 6 chickens for 3 dollars 37½ cents; what is that apiece? Ans. 6¼ cents.

5. Paid for 12 bushels of apples, 3 dollars 75 cents; how much is that a bushel? Ans. 31¼ cents.

6. Sold 34 sheep at auction, for 38 dollars 25 cents; what does the sheep average? Ans. 1 dol. 12½ cts.

7. Planted 420 apple-trees in 7 rows; how many in each row? Ans. 60.

8. Paid for the above apple-trees, 37 dollars 80 cents; what is that for each apple-tree? Ans. 9 cents.

9. Paid a man for working for me 24 days, the sum of 15 dollars; what is that per day? Ans. 62½ cents.

10. A man travelled from New-York to this town, (Washington, G. C. Ohio,) in 23 days. How much did he travel each day, allowing the distance to be 460 miles?

Ans. 20 miles.

Division of divers denominations.

RULE. In dividing into the left hand, or greater division of the given sum to be divided, divide as in whole numbers, and reduce the remainder, (if any,) to the next lowest denomination, adding to it the number of the same denomination in the given sum, and divide this in like manner; and so proceed till all are divided. If any remain after the last division is obtained, that remainder is of the same name as the dividend or denomination that was last divided.

EXAMPLES.

oz.	*dwt.*	*grs.*
4)5	18	12
1	9	15

Here I give one to shew more clearly how you are to proceed with the following. First, I say, how often is 4 contained in 5, which is 1, and 1 over. The 1 that is over is 1 ounce, which you must reduce to the next lowest denomination, which is pennyweights. By having the table of Troy weight committed to memory, ere you come this far, you will find 1 ounce contains 20 pennyweights, to which you add the 18 dwts. in the given sum, making 38; and divided by 4, the quotient is 9, which I set down in the place of pennyweights, and there is 2 over, which 2 is pennyweights. By the table, each pennyweight contains 24 grs. making 48 grains in the 2 dwts. and the 12 grs. in the given sum, will make 60 grains. Then I seek how often 4 will go into 6, (the first figure of the grains,) and find the quotient is 1, which I set in grains, and 2 over, which 2, with the cipher annexed to 6, make 20. Then I say, how often will 4 go into 20, and find the quotient 5, which I set down on the right hand of the last 1, making the quotient in all, 1 oz. 9 dwt. 15 grs.

TROY WEIGHT.

oz. dwt. grs.	oz. dwt. grs.	oz. dwt. grs.
5)16 16 20	8)70 19 16	9)64 14 22

AVOIRDUPOIS WEIGHT.

lbs. oz. drs.	cwt. qr lbs. oz.	T. cwt. qr. lbs.
8)24 14 12	7)18 2 16 14	6)68 14 3 14

APOTHECARIES' WEIGHT.

dr. sc. gr.	lbs. oz. dr. sc. gr.	lbs. oz. dr. sc. gr.
10)6 1 18	6)16 8 5 2 14	8)24 6 7 2 12

CLOTH MEASURE.

yds. qr. na.	E E. qr. na.	E. Fl. qr. na.
4)28 3 2	5)64 4 3	8)79 2 3

DRY MEASURE.

pk. qt pt.	bu. pk. qt.	bu. pk. qt. pt.
4)6 6 1	8)120 3 7	6)370 2 7 1

LIQUID MEASURE.

P. hhd. gal.	tun. p. hhd. gal.	tun. hhd. ga. qt. pt. gill.
8)24 1 60	6)64 1 1 54	12)84 1 38 3 1 1

LONG MEASURE.

lea. m fur.	mi. fur. pole.	lea. mi. fur. ft. inc.
6)16 2 6	8)84 6 32	12)160 2 7 200 9

LAND MEASURE.

A. R. P.	A. R. P.	A. R. P.
12)84 3 24	8)160 2 24	9)260 3 32

SOLID MEASURE.

ton. feet.	ton. feet.	cord. feet.
4)16 46 hewn.	8)24 36 round.	8)264 120

TIME.

yrs.	mo.	w.	da.
6)64	8	3	4

da.	hr.	min.
8)324	16	54

week.	da.	hr.	min.	sec.
12)764	4	21	16	30

CIRCULAR MOTION.

sign.	°	′
6)64	10	55

°	′	″
12)360	56	32

sign.	°	′	″
8)360	24	52	32

REDUCTION

Teaches to bring or change numbers of a higher denomination to that of a lower, yet retaining the same value.

Rule. Multiply the highest denomination given, by so many of the next less, as make one of that greater; and this continue, till you have brought it down as low as your question requires. But if there be smaller denominations attached to the greater, as 3 cwt. 2 qrs. 14 lbs. you must, in your multiplying, add in the denomination that is of the same name of that which you are reducing the higher one to, and so on, as for example:

	cwt.	*qrs.*	*lbs.*
	3	2	14
	4		
	14 qrs.		
	28		
	116		
	29		
Ans.	406 lbs.		

Reduction ascending, (improperly called,) is the bringing a lower denomination to a greater, yet retaining the same value.

Rule. Divide the lowest denomination given, by as many of the same name as make one of the next greater or higher, and so on through all the denominations, as far as your question requires. But if there happens to be a remainder, after your first divisor is divided, that remainder is of the same name as the dividend, before it was divided; and all the remainders, throughout the operation of the

question, will be of the same name as the dividend, before it was divided. For example, bring 406 lbs. to *cwt. qrs. lbs.* Here, 28 lbs. make a quarter, which I divide, because a quarter is the next highest denomination, and the quotient is 14, and 14 remains, which 14 is pounds, as the dividend was pounds, before it was divided by 28. Now, the quotient 14 is quarters, therefore the remainder, after that is divided, will be quarters. Here again I divide by 4, the next greatest denomination, as 4 qrs. make 1 cwt. and there remains 2, which 2 is quarters, and the quotient 3 cwt. See the operation.

lbs.	*qrs.*
28)406(14 qrs.	4)14(3 cwt.
28	12
126	2 qrs.
112	
14 lbs.	Ans. 3 cwt. 2 qrs. 14 lbs.

Questions.

1. Reduce 24 eagles 8 dollars 6 dimes 8 cents 7 mills, to mills. Ans. 248687 mills.
2. Bring 248687 mills to eagles, dollars, &c. Ans. 24 eag. 8 dolls. 6 dim. 8 cts. 7 m.
3. Reduce 764 dollars 75 cents to cents. Ans. 76475 cents.
4. Bring 76475 cents to dollars. Ans. 764 dollars 75 cents.
5. Reduce 800 dolls. to mills. Ans. 800000 mills.
6. Bring 800000 mills to eagles. Ans. 80 eagles.
7. Reduce 64 dollars 12½ cents to mills. Ans. 64125 mills.
8. Bring 64125 mills to cents. Ans. 6412½ cents.
9. Bring 6412½ cents to dollars. Ans. 64 dollars 12½ cents.
10. Reduce 380 dollars 12½ cents to mills. Ans. 380125 mills.
11. Bring 6543794 mills to dollars. Ans. 6543 dolls. 79 cts. 4 m.
12. Reduce 719 dollars 62½ cents to mills. Ans. 719625 mills.
13. Bring 719625 mills to dollars. Ans. 719 dolls. 62½ cts.

NOTE. ½ cent is 5 mills. See table of money, where 10 mills make 1 cent; consequently, 5 mills make ½ cent.

TROY WEIGHT.

1. Reduce 47 lbs. 10 oz. to grains. Ans. 275520 grs.
2. Bring 275520 grs. to pounds. Ans. 47 lbs. 10 oz.
3. In 10 pounds of silver, how many spoons, each 5 oz. 10 dwts.? Ans. 21 spoons, and 90 dwts. over.
4. Reduce 20 pounds of silver to grains. Ans. 115200 grains.
5. Bring 115200 grs. to pounds. Ans. 20 lbs.
6. In 4560 grains, how many teaspoons, each half an ounce? Ans. 19 teaspoons.
7. In 19 teaspoons, each half an ounce, how many grains? Ans. 4560 grains.
8. In 17 ingots of silver, each 27 oz. 10 dwts. how many grains? Ans. 224400 grains.

AVOIRDUPOIS WEIGHT.

1. Reduce 7 cwt. 2 qrs. 17 lbs. to pounds. Ans. 857 lbs.
2. Bring 857 lbs. to cwt. Ans. 7 cwt. 2 qrs. 17 lbs.
3. Reduce 7 cwt. 3 qrs. 10 lbs. to pounds. Ans. 878 lbs.
4. In 14048 ounces, how many hundred weight? Ans. 7 cwt. 3 qrs. 10 lbs.
5. Bring 3 tons to pounds. Ans. 6720 lbs.
6. Bring 6720 lbs. to cwt. Ans. 60 cwt.
7. Reduce 60 cwt. to quarters. Ans. 240 qrs.
8. In 3 lbs. of cloves, how many parcels, each 12 oz.? Ans. 4 parcels.
9. In 400 lbs. of soap, how many parcels, each 5 lbs.? Ans. 80 parcels.
10. In 4 loads of hay, each 5 cwt. 1 qr. 10 lbs. how many pounds? Ans. 2392 lbs.
11. Bring 2392 pounds to tons. Ans. 1 ton 1 cwt. 1 qr. 12 lbs.

APOTHECARIES' WEIGHT.

1. Reduce 14 lbs. 8 oz. 4 drs. 2 scr. 16 grs. to grains. Ans. 84776 grains.
2. Bring 84776 grains to pounds. Ans. 14 lbs. 8 oz. 4 drs. 2 scr. 16 grs.
3. Reduce 5 pounds of medicine to grains. Ans. 28800 grains.

CLOTH MEASURE.

1. Reduce 26 yds. 2 qrs. to quarters. Ans. 106 qrs.
2. Bring 106 quarters to yards. Ans. 26 yds. 2 qrs.

3. Reduce 52 yards of Irish linen to nails.
Ans. 832 nails.
4. Bring 832 nails to yards. Ans. 52 yards.
5. How many shirts, of 2 yards 3 quarters each, can be taken out of a piece of Irish linen, containing 52 yards 1 quarter? Ans. 19 shirts.
6. In 19 shirts, each 2 yards 3 quarters, how many yards?
Ans. 52 yards 1 quarter.
7. In 4 pieces of muslin, each measuring 46 yards, how many ells English? Ans. 147 ells, and 1 qr. over.
8. In 416 ells Flemish, how many yards?
Ans. 312 yards.
9. In 846 nails, how many ells French?
Ans. 35 ells 1 qr. 2 na.

LONG MEASURE.

1. Reduce 49 yards to feet. Ans. 147 feet.
2. Reduce 64 miles 2 furlongs to furlongs.
Ans. 514 furlongs.
3. Bring 514 furlongs to miles.
Ans. 64 miles 2 furlongs.
4. Reduce 24 miles 7 furlongs 30 perches 2 yards 1 foot 8 inches, to barleycorns. Ans. 4746336 barleycorns.

DRY MEASURE.

1. Reduce 742 bushels 2 pecks to pecks.
Ans. 2970 pecks.
2. Bring 2970 pecks to bushels.
Ans. 742 bushels 2 pecks.
3. Reduce 49 bushels 3 pecks 5 quarts, to quarts.
Ans. 1597 quarts.
4. Bring 1597 quarts to bushels.
Ans. 49 bushels 3 pecks 5 quarts.
5. Reduce 64 bushels 3 pecks 1 gallon, to quarts.
Ans. 2076 quarts.
6. Bring 2076 quarts to bushels.
Ans. 64 bushels 3 pecks 1 gallon.

LIQUID MEASURE.

1. In 9 tuns of wine, how many hogsheads, gallons and quarts? Ans. 36 hhds. 2268 gal. 9072 qts.
2. Bring 9072 quarts to gallons, hogsheads and tuns.
Ans. 2268 gal. 36 hhds. 9 tuns.
3. Reduce 24 hhds. 18 gal. 1 qt. to pints.
Ans. 12242 pints.

4. Bring 12242 pints to hogsheads.
Ans. 24 hhds. 18 gal. 1 qt.

5. How many pints, quarts, and half gallons, each an equal number, can be taken out of a pipe of wine?
Ans. 1 gross, or 144.

6. How many bottles, containing a pint and a half each, can be taken out of a barrel of cider?* Ans. 168 bottles.

LAND MEASURE.

1. Reduce 341 acres 3 roods 25 poles, to poles.
Ans. 54705 poles.

2. Bring 54705 poles to acres. Ans. 341 a. 3 r. 25 p.

3. If a piece of land contains 94 acres, and I want to sell 24 acres 2 roods 20 perches, how many square perches will remain? Ans. 11100 perches.

4. Three fields contain, the first, 8 acres, the second, 7 acres, and the third, 14 acres 1 rood. How many shares can be taken out of it, 76 poles each share?
Ans. 61 shares, and 44 roods over.

SOLID MEASURE.

1. Reduce 14 tons of hewn timber to inches.
Ans. 1209600 inches.

2. Bring 1209600 inches of hewn timber to tons.
Ans. 14 tons.

3. Reduce 19 tons of round timber to inches.
Ans. 1313280 inches.

4. Bring 1313280 inches of round timber to tons.
Ans. 19 tons.

5. Reduce 20 cords of wood to solid feet.
Ans. 2560 feet.

6. Bring 2560 solid feet to cords. Ans. 20 cords.

7. How many solid feet in a grindstone, 4 feet in diameter, and 18 inches thick?† Ans. 18 feet.

8. How many solid feet in a millstone that is 5 feet in diameter, and 22 inches thick? Ans. 34 ft. 648 inc.

TIME.

1. Reduce 6 hours to minutes. Ans. 360 minutes.

2. In 7 days how many minutes? Ans. 10080 min.

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* A barrel of cider is 32 gallons.

† For mill or grindstones, to the given diameter add half that diameter, and multiply that sum by the said half diameter, and multiply that product by the thickness, and that product, divided by 1728, will give the contents in feet; and if there be a remainder, it will be inches.

3. In 41 weeks how many seconds?
Ans. 24796800 seconds.
4. In 24796800 seconds, how many hours?
Ans. 6888 hours.
5. In 184009 minutes, how many days?
Ans. 127 days 18 hours 49 minutes.
6. In 127 days 18 hours 49 minutes, how many minutes?
Ans. 184009 minutes.
7. How many seconds in a year, allowing the year to consist of 365 days 6 hours. Ans. 31557600 seconds.
8. From March 2nd, to November 19th following, inclusive, how many days? Ans. 262 days.
9. How many weeks in 262 days?
Ans. 37 weeks 3 days.
10. How many seconds in a week?
Ans. 604800 seconds.

CIRCULAR MOTION.

1. Reduce 7 signs 15° 24′ 40″ to seconds.
Ans. 811480 seconds.
2. Bring 811480 min. to degrees. Ans. 13524° 40′
3. In 974827 minutes, how many degrees and signs?
Ans. 16238° 47′ and 541 signs 8° 47′

Questions to exercise the foregoing.

1. Bought 320 bushels of oats for 80 dollars; what did the oats stand me a bushel? Ans. 25 cents.
2. Lent my friend 3964 dollars, and received from him, at one time, 200 dolls. 62½ cts. at another time, 409 dolls. 18¾ cts. and at a third time, 514 dolls. 87½ cts. and a draft on the bank for the balance. How much was the draft drawn for? Ans. 2839 dolls. 31¼ cts.
3. Paid in one week the following sums of money: For corn, 96 dolls. 75 cts. for wheat, 112 dolls. 12½ cts. for flour, 62 dollars 37½ cents, and for sundry small articles, 2 eagles 3 dimes 5 mills. How much did I pay in all?
Ans. 291 dolls. 55½ cts.
4. Bought 8 cows for 7 dolls. 75 cts. each; paid for pasturing them, 1 dollar 6¼ cents a head, and for corn, 5 dollars 18¾ cents; and then sold them at 11 dollars 3 dimes apiece. How much did they stand me, and how much did I receive for them, and what did I gain by the transaction?

Ans. { Stood me $75 68¾ cts. / I received 90. 40 / I gained 14 71¼

5. If I spend 3 dimes 5 mills a day, for pocket expenses, and drink a bottle of wine each day, which costs 93¾ cents a bottle, and lay out, for house expenses, 2 dollars 6¼ cents each day, how much are my expenses at the end of a year?
Ans. 1209 dolls. 6¼ cts.

6. Sold 15 pieces of cloth to a French merchant, each piece containing 20 yards. How many ells French were in the 15 pieces? Ans. 200 ells.

7. In 10 bales of cloth, each bale 12 pieces, and in each piece 25 ells Flemish, how many yards? Ans. 2250 yds.

8. The sun travels through six signs of the zodiac in six months; how many degrees, minutes and seconds, is that?
Ans. 180 deg. 10800 min. 648000 sec.

9. Bought 4 loads of hay, weight as follows: First load weighed 9 cwt. 3 qrs. 6 lbs. the second, 8 cwt. 1 qr. 14 lbs. the third, 7 cwt. 3 qrs. 22 lbs. and the fourth, 8 cwt. 1 qr. 14 lbs. for 2 dolls. 3 dim. 7 cts. 5 m. a load. What is the whole weight of hay, and what is the amount?
Ans. 1 ton 14 cwt. 2 qrs. and cost 9 dolls. 50 cts.

10. Sold 200 pairs of men's shoes for 225 dolls. and 150 pairs of women's shoes for 131 dolls. 2 dim. 5 cts. and 100 pairs of boys' shoes for 6 eagles 250 cents. What was the price of the men's, women's, and boys' shoes a pair?

Ans. { Men's cost $1 25 cts. a pair.
Women's cost 87½
Boys' cost 62½

11. How many times will a ship, that is 97 feet 6 inches long, sail her length, in the distance of 12800 leagues 10 yards? Ans. 2079508 times.

12. The sun is 95 millions of miles from the earth; and a cannon-ball, at its first discharge, flies about a mile in 7½ seconds. How long would a cannon-ball be, at that rate, in flying from the sun? Ans. 22 yrs. 216 da. 12 hrs. 40 min.

13. How many strokes does a regular clock strike in a year? Ans. 56940.

14. How long will it take to count a million, at fifty a minute? Ans. 13 da. 21 hrs. 20 min.

15. What is the difference between six dozen dozen, and half a dozen dozen? Ans. 792.

16. The forewheels of a wagon are 14 feet 6 inches, and the hinder wheels 15 feet 9 inches in circumference. How many more times will the forewheels turn round, than the hindwheels, in running 248 miles? Ans. 7167 times.

17. Four merchants put into the bank, the sum of 14884 dollars 75 cents; how much did each put in?
Ans. 3721 dolls. 18¾ cts.

18. A merchant bought 4 hhds. of tobacco, which weighed in all, 35 cwt. 3 qrs. at 5 cts. per pound. Sorting and packing stood him 5 mills a pound, and carriage to Baltimore, 2 cents 5 mills a pound. How much does the tobacco stand in all, and how much per pound?

Ans. 320 dolls. 32 cts. in all, and 8 cts. per lb.

19. How long would it take a man to travel round the earth, (allowing no obstacles,) at the rate of 25 miles a day, without intermission? Ans. 864 days.

MECHANICS' AND FARMERS'

ARITHMETIC AND ACCOUNTS.

MANY there be, who cannot get much education, such as apprentices, and farmers' sons. When I have such under my tuition, I always adopt this arithmetic and accounts, before they get as far as Division, owing entirely to the length of time they are to remain at school. There are many of my scholars in this county, (Guernsey, Ohio,) who never had more than eight weeks ciphering, under my tuition, who can and do keep their accounts, and write promissory notes correctly. Knowing it to be useful and necessary to them, I always practise it, independent of any other dictates, than those of my conscience.

The following will exercise the scholar in what he has gone through in part.

A Tailor's Account.

Mr. RICHARD PAYWELL

To JOHN COYLE, Dr.

1827.			$	cts.
May	6	To making a great coat	4	25
"	17	" making a suit of clothes for your son	3	75
"	27	" making a surtout for yourself	3	93¾
"	31	" making a pair of pantaloons	2	00
June	1	" making a body coat	5	75
"	8	" making a vest	1	12½
"	"	" expended of my own thread and silk	0	50
		Due me	$	

NOTE. Mr. signifies *Mister*, or *Master*, and Dr. *debtor*. The Mr. is generally placed before the debtor's name, and Dr. after the creditor's name.

A Shoemaker's Account.

Mr. Thomas Hanna

To Daniel Bumgarner, Dr.

1827.			$	cts.
May	2	To 1 pair of boots	5	00
"	5	" 1 pair of shoes	2	00
"	7	" new feet to a pair of boots	2	25
"	9	" vamping and soaling a pair of shoes	0	87½
"	20	" 2 pairs of shoes for your wife	3	00
"	24	" 1 pair of shoes for James	1	00
"	26	" mending a pair of shoes for John	0	18¾
"	31	" 1 pair of shoes for George	1	50
June	1	" mending 4 pairs of shoes	1	12½
"	4	" making a pair shoes for self, you finding the leather	0	75
"	13	" 1 pair of boots for Charles	3	12½
"	15	" mending 2 pairs of shoes, and a pair of boots	1	87½
"	18	" 1 pair of boots for George	2	62½
"	25	" 3 pairs of coarse shoes for workmen	4	50
		Due me	$	

A Tanner's Account.

Mr. John Cash

To Richard Leatherall, Dr.

1827.			$	cts.
May	6	To half a side of soal leather, weight 24 lbs. at 31¼ cts. per lb.		
"	8	" a quarter kipskin	0	87½
"	14	" 14 lbs. soal leather, at 30 cts. per lb.		
"	18	" half a calfskin	1	62½
"	26	" leather for a pair of boots	2	81¼
"	30	" upper leather for a pair of shoes	0	50
June	6	" harness leather	3	12½
"	9	" 2 calfskins	2	87½
"	12	" 2 pairs of half soals, per your order	0	31¼
"	16	" a quarter kipskin.	1	12½
		Due me	$	

A Saddler's Account.

Mr. Thomas Upright

To William Skinner, Dr.

1827.			$	cts.
May	6	To a new saddle for yourself	16	00
"	12	" a new saddle for your wife	20	00
"	18	" mending 2 old saddles	1	62½
"	26	" 4 working, or blind bridles, at $1 62½ cts. each		
"	28	" 2 new collars, at $1 37½ cts. each		
June	1	" mending 2 old collars	0	87½
"	6	" 2 new bridles, at $1 87½ cts. each		
"	8	" mending 2 old bridles	0	62½
"	14	" 2 new girths and a surcingle	1	31¼
"	16	" mending saddlebags	0	18¾
"	20	" a wagonwhip	0	75
		Due me	$	

A Blacksmith's Account.

Mr. George Moneyman

To Jacob Barton, Dr.

1827.			$	cts.
May	4	To shoeing 2 horses	2	12½
"	8	" 4 removes	0	25
"	10	" mending a logchain	0	62½
"	16	" sharpening plough-irons	0	31¼
"	24	" making an axe, iron and steel yours	1	50
"	28	" repairing an old axe	0	37½
June	1	" repairing a scythe	0	12½
"	4	" steeling and tempering mason tools	1	87½
"	8	" putting iron hoops on a well-bucket	1	06¼
"	"	" making a chain for the same	0	50
"	14	" mending drawing-chains	0	87½
		Due me	$	

A Wheelwright's Account.

Mr. Thomas Readypay

To John M'Mullin, Dr.

1827.			$	cts.
May	5	To 4 spinning wheels, at $3 75 cts. each		
"	8	" 3 doz. weaver's spools 1 12 per doz.		
"	9	" mending 6 chairs, at 31¼ cts. each		
"	21	" 2 new flyers, and teething a pair	0	87½
"	28	" a new woollen wheel	2	62½
June	1	" a new check reel	1	50
"	12	" repairing an old wheel and reel	0	75
		Due me	$	

A Carpenter's Account.

Mr. William Banker

To William Robinson, Dr.

1827.			$	cts.
May	4	To laying 650 feet of flooring, at $1 25 cts. per 100 ft.		
"	8	" making, hanging, and fitting 3 doors	10	93¾
"	20	" making 14 twenty-four light sashes, at 6¼ cts. a light		
"	24	" partitioning a room off the kitchen	3	87½
"	26	" making a door for said room	2	12½
June	1	" making a kitchen table	2	31¼
"	12	" shingling your new house	56	87½
		Due me	$	

A Laborer's Account.

Mr. Thomas Plenty

To John Needy, Dr.

1827.			$	cts.
May	5	To 4 days chopping wood, at 37½ cts. per day		
"	9	" 5 days ploughing corn ground, at 50 cts. per day		
"	16	" 6 days thrashing wheat, 56¼ cts. a day		
"	26	" 5 days planting corn 31¼		
June	1	" 3 days driving wagon 56¼		
"	6	" 5 days attending masons 37½		
"	26	" 8 days mowing hay 56¼		
		Due me	$	

A Farmer's Account.

Mr. JONATHAN PAYMONEY

To JOHN FARMER, Dr.

1827.			$	cts.
May	4	To 4 bushels corn, at 31¼ cts. per bushel		
"	9	" 6 bush. potatoes 25 "		
"	12	" 8 pounds butter, at 6¼ cts. per lb.		
"	13	" 10 dozen of eggs, at 4 cts. per dozen		
"	14	" 2 tons 5 cwt. of hay, at 25 cts. per cwt.		
"	16	" 100 pounds of flour	1	50
"	24	" 8 dozen of chickens, at 6¼ cts. apiece		
"	26	" cabbage and beets	1	62½
June	1	" 24 pounds hog's lard, at 8 cts. per lb.		
"	6	" 12 gallons soap, at 18¾ cents per gal.		
		Due me $		

Another.

Mr. TIMOTHY GOODPAY

To ROBERT INDUSTRY, Dr.

1827.			$	cts.
May	2	To 4 days ploughing, at $1 25 cts. a day		
"	8	" 3 days hauling wood 1 12½ "		
"	14	" 6 tons of hay, at 18¾ cts. per cwt.		
"	20	" a fat cow	12	00
"	26	" a veal calf	3	06¼
"	29	" 4 fat sheep, at $2 6¼ cts. apiece		
June	4	" a two-year-old colt	17	50
"	8	" 5 days hauling stone, at $1 per day		
"	16	" 2 days hauling tobacco	2	00
		Due me $		

The learners are to observe, in drawing off or furnishing their accounts, that the two dots, thus, (") under May, and under To, is a clerical and mercantile style of repeating the word, under which those dots are placed, and signifies that May is continued on, until the month June is wrote; and the two dots under To, are to be continued as far as the accounts run. The two dots signify, that May, June, and To. are made mention of, as it would appear unnecessary to repeat those words at full length, as often as is required.

Examples of Promissory Notes, Receipts and Orders.

On demand I promise to pay Mr. John Moneyless, or order, the sum of fifty dollars, for value received this 9th day of June, 1827. THOMAS DEBTOR.

Six months after date, I promise to pay Mr. Thomas Goodman, or order, the sum of twenty dollars and fifty cents, for value received this 9th day of June, 1827.

JAMES CREDIT.

A Joint Note.

Six months after date, we jointly or severally promise to pay Mr. George Trusty, or order, the sum of 10 dollars 37½ cents, for value received this 9th day of June, 1827.

JOHN NEEDY,
HENRY DITTO.

A Receipt in part payment.

Received of Thomas Tenant, the sum of thirty dollars, on account, this 16th day of June, 1827.

JOHN RECEIVER.

A Receipt in full.

Received of David Ballantine, the sum of five hundred dollars, in full of all demands whatever to this date, June 9th, 1827. THOMAS MONEYMAN.

Orders.

Mr. JOHN LENDER,

SIR—Please lend me your wagon a few days, and I will return you the like favour another time, if in my power.

Yours, &c.

June 16th, 1827. THOMAS BORROWER.

Mr. HENRY H. EVANS,

SIR—Please send me by the bearer, (John Carefull,) twenty pounds of iron, and two pounds of steel, and I will pay you the next time I go to town, which will be in a few weeks hence. Yours respectfully.

THOMAS CRAVER.

Monday, 4th June, 1827.

NOTE. When you draw a promissory note, be sure you always write, *or order*, otherwise the note is not transferable, as no person can recover it by law, but the person in whose favour it is drawn. When the words, *on account*, are written in a receipt, they shew that the money then paid, is only a part of the debt. *Orders* may be drawn in many forms, according to the nature of the case; but in all cases they should be briefly drawn.

SECTION II.

THE GOLDEN RULE, OR RULE OF THREE DIRECT.

This Rule has three numbers given to find a fourth, which shall bear the same proportion to the third, as the second does to the first. Two of the given numbers in your question are always of the same name or kind, and the other is of the same name of the fourth, or number sought, which is the answer. The first number is often known by the word *If*, or *At*. The third is a demand, and is known by the words, *What will*, *What cost*, *How much*, *How far*, *&c.* The first and third words are always placed before the first and third numbers, in the given question.

RULE. Write the word If, or At, down first, and place your numbers to the right hand of If, or At, so that the first and third numbers will be of the same name or kind, and the second number of the same name as the answer. Bring the first and third numbers, or terms, to the least denomination mentioned in either, and the second term or number to the least denomination in it. Then multiply the second and third numbers together, and divide that product by the first term or number, and the quotient will be the same name or denomination you left the second in, and is the answer to the question, which is the fourth term, or number sought, and may be reduced to a higher name, yet retaining the same value. Or, divide the second number by the first, and multiply the quotient by the third, and the product will be the answer. Or, divide the third number by the first, and multiply the quotient by the second, and the product will be the answer. Or, divide the first number by the third, and the second by that quotient, and the last quotient will be the answer.

EXAMPLES.

1. If 8 cents be paid for a pound of sugar, how much must I pay for 112 lbs. at that rate? Ans. 8 dolls. 96 cts.

	lb.		*cts.*		*lbs.*
If	1	:	8	: :	112
					8
Ans.					896 cts. Or 8 dolls. 96 cts.

2. If 8 dolls. 96 cts. be paid for 112 lbs. of sugar, what is that a pound, at that rate? Ans. 8 cents.

3. If I paid 8 dollars 96 cents for 112 lbs. of sugar, what quantity can I get for 8 cents, at that rate? Ans. 1 lb.

4. How much corn can I get for 100 dollars, at the rate of 25 cents a bushel? Ans. 400 bushels.

5. Paid 100 dollars for 400 bushels of corn; what is that a bushel? Ans. 25 cents.

6. Bought 400 bush. of corn for 100 dolls. what quantity can I get, at that rate, for 2 dimes 5 cents?

Ans. 1 bushel.

7. If I work for 50 cents a day, how much is my wages at the end of 6 months, (deducting 26 days for Sundays,) allowing the 6 months to be 183 days?

Ans. 78 dolls. 50 cts.

8. Paid 51 dolls. 6 cts. for a flock of sheep, at 1 dol. 11cts. a head; how many did I buy? Ans. 46.

9. Bought 3 hhds. of sugar, each weighing 8 cwt. 1 qr. 12 lbs. at the rate of 7 dollars 26 cents per cwt. What did I pay? Ans. 182 dolls. 1 ct. 8 m.

10. What will 4 cwt. 2 qrs. 8 lbs. of iron come to, at 48 cents for 4 pounds? Ans. 61 dolls. 44 cts.

11. What will 396 lbs. of pork come to, at 2 cts. a pound?

Ans. 7 dolls. 92 cts.

12. If 20 bushels of oats last 4 horses 1 week, how many weeks will 100 bushels last them, at that rate?

Ans. 5 weeks.

13. Bought a piece of cloth for 33 dolls. at 75 cts. a yard; how many yards were in the piece? Ans. 44 yards.

14. A merchant bought 14 pipes of wine, and is allowed six months credit, but for ready money, gets it 8 cents a gallon cheaper. How much did he save by paying the ready money? Ans. 141 dolls. 12 cts.

15. When sugar is sold for 12 dolls. 32 cts. per cwt. how much can I get, at that rate, for 6 dollars 16 cents?

Ans. 56 lbs.

16. If 3 and 4 make 9, what will 8 and 6 make?

Ans. 18.

17. A loaded wagon was sent by Mr. Rabe to Wheeling, distant 40 miles, who arrived at Wheeling 16 hours after his departure from Washington. At what rate per hour did he travel? Ans. 2 miles 4 furlongs.

18. The same wagon returned empty in 10 hrs. 40 min. at what rate per hour did he travel?

Ans. 3 miles 6 furlongs.

19. If a family of 10 persons spend 3 bushels of wheat in a month, how many bushels will serve them, when they are 30 in family? Ans. 9 bushels.

20. What is cheese per cwt. when a pound costs 12½ cts.?

Ans. 14 dollars.

21. If lead is sold at 5 cts. a pound, what is 3 cwt. worth, at that rate? Ans. 16 dolls. 80 cts.

22. If 100 dollars gain 6 dollars interest in a year, how much will 49 dollars gain in the same time?

Ans. 2 dollars 94 cents.

23. How many reams of paper, at 1 dollar 6 dimes 6 cts. 1 dollar 9 dimes 7 cents, and 23 dimes 1 cent per ream, may I get for 1057 dolls. 32 cts. and of each an equal number?

Ans. 178 reams.

24. When iron is sold for 67 dollars 20 cents a ton, what is that a pound? Ans. 3 cents.

25. How much will a grindstone, 4 ft. 9 inc. in diameter, and 9 inches thick, come to, at 1 dol. 10 cts. per solid foot?

Ans. 13 dolls. 96 cts. +

26. What will 18 cwt. 3 qrs. 14 lbs. of tobacco come to, at 6 dollars the hundred pounds?

Ans. 126 dollars 84 cents.

27. If a man's yearly income be 1186 dolls. 25 cts. how much is it per week? Ans. 22 dolls. 81¼ cts.

28. If a man spends 3 dollars 25 cents a day, and saves 496 dollars 50 cents at the end of the year, how much is his yearly income? Ans. 1682 dolls. 75 cts.

29. The distance from here to Cambridge is 10 miles; now, how many steps will a man make in going to Cambridge, allowing he gains 18 inches each step; and how long will he be in going there, supposing he makes 2 steps in 3 seconds?

Ans. { 35200 steps.
Length of time, 14 hrs. 40 min.

30. I want to know the breadth of a river, running by the side of a castle, whose height I found to be 60 feet, from the surface of the water; and having a stick in my hand, just 3 feet long, I set it up perpendicular, and it cast a shadow at the same time of 5 feet. Now, the shadow of the steeple was 21 feet on the land, from the water's edge, where I was. What is the breadth of the river? Ans. 79 feet.

31. If 60 gallons of water in an hour fall into a cistern, containing 300 gallons, and by a pipe in the cistern, 30 gallons run out in an hour, in what time will it be filled?

Ans. 10 hours.

32. A and B depart from the same place, and go the same way; but A goes 5 days before B, at the rate of 20 miles a day; and B follows, at the rate of 25 miles a day. What distance must he travel, that he may overtake him?

Ans. 500 miles.

33. If the schooltax in this state be 5 mills on a dollar, what must that man pay, who is taxed 820 dollars 50 cents?

34. How many more revolutions will the smaller wheels of a wagon make than the greater, in going a mile, the circumference of the larger ones 12 feet, and the lesser ones 9 feet? Ans. 146⅔ more.

35. Bought a pipe of wine for 84 dollars, and found it had leaked 12 gallons. I sold the remainder at 12 cents 5 mills a pint. Did I save myself? Ans. I gained 30 dolls.

36. Two men depart from the same place at the same moment, and one goes due west 30 miles a day, the other due east 25 miles a day. How far are they apart the third day after their departure? Ans. 165 miles.

37. How much corn, at 31¼ cents a bushel, must I give for 150 bushels of wheat, at 56¼ cents a bushel?
Ans. 270 bushels.

38. What will 1 mile 6 furlongs 25 rods of the making of the turnpike road come to, at 2 dollars 3 dimes, and 250 mills a rod? Ans. 1491 dolls. 75 cts.

39. If 250 men in a garrison consume 54 barrels of pork in six months, how many barrels will a regiment of 1000 strong consume in the same time? Ans. 216 barrels.

40. If 1 yard 2 quarters of cloth cost 7 dollars, what will an ell Flemish cost, at that rate? Ans. 8 dolls. 50 cts.

41. If a man's yearly income be 1000 dolls. and he spends 1 dol. 62½ cts. a day, how much does he save at the year's end? Ans. 406 dolls. 87½ cts.

42. If 2000 dollars were paid for 20 pieces of cloth, at the rate of 12 dollars 50 cents for every five yards, how many yards were in each piece, allowing them to measure alike?
Ans. 40 yards.

43. What is the price of four pieces of cloth, containing 24, 26, 27 and 31 yards, at 2 dolls. 25 cts. the ell English?
Ans. 194 dolls. 40 cts.

44. Bought three bales of gloves, the first bale contained 8 dozen dozen and 10 pairs, the second contained 7 score and 7 pairs, and the third contained a quarter of a dozen dozen and two pairs, at 25 cts. a pair. How much do they come to, and how many pairs had I?

Ans. { Cost $336 75 cts.
{ 1347 pairs.

45. A merchant in Baltimore failed in trade, and owed me, at that time, 2500 dollars, for tobacco I sold him; but he sent me word, that his effects and outstanding debts, when sold and collected, will enable him to pay his creditors 75 cts. per dollar. How much then am I to receive for my tobacco debt, and how much do I lose by the merchant failing? Ans. I receive $1875, and lose $625.

46. A sets out from a certain place, and goes at the rate of 20 miles a day. Five days after, B sets out from the same place, and goes at the rate of 25 miles a day the same road. In how many days will he overtake A?

Ans. 20 days.

47. If the earth, which is 360 degrees in circumference, turns round once in 24 hours, how far are the inhabitants carried round in an hour living at the equator, a degree there being 69½ miles? Ans. 1642 miles 4 furlongs.

48. Bought 3 hhds. of peach brandy, containing 60 gallons 3 quarts, 61 gallons 2 quarts, 59 gallons 1 pint, and 63 gallons 3 quarts 1 pint, at 6 dimes 40 cents for every 4 gallons. What is the whole amount? Ans. 61 dolls. 31¼ cts.

49. How much will a quarter section of land come to now, the congress price per acre being only 1 dollar 25 cents?

Ans. 200 dollars.

50. If a man lays out 121 dollars 23 cents in tobacco, and thereby gains 39 dollars 51 cents, how much will he gain on 100 lbs. weight, which cost 6 dolls.? Ans. 1 dol. 95½ cts.

THE SINGLE RULE OF THREE INVERSE.

This Rule teaches, by having three numbers given, to find a fourth, which will have the same proportion to the second, as the first has to the third. When more requires less, or less requires more, the question is in this Rule; but when more requires more, and less requires less, the question belongs to the Rule of Three Direct.

RULE. State your question as in the Rule of Three Direct, and multiply the first and second terms together; then divide the product by the third term, and the quotient will be the answer, in the same name as the second number.

EXAMPLES.

1. If 6 men can mow a meadow in 24 days, how many can mow the same in 4 days? Ans. 36 men.

days.	*men.*	*days.*
If 24	: 6	:: 4

6

4)144

—36 men. Ans.

NOTE. The product of the third number, multiplied by the answer, will be always equal to the product of the first multiplied by the second, in Inverse Proportion; and the first number, multiplied by the answer, will be equal to the second multiplied by the third, in Direct Proportion.

2. If 48 men build a wall in 24 days, how many men can do the same in 192 days? Ans. 6 men.

3. If 6 men can build a wall in 192 days, how many men will build the same in 24 days? Ans. 48.

4. If 6 men, in 192 days, can build a wall, how many days will it take 48 men to do the like work? Ans. 24 days.

5. In 24 days 48 men can build a wall, how many days will 6 men be doing the same work? Ans. 192 days.

6. If I lend my friend 2000 dollars for 6 months, how long ought he to lend me 500 dolls. to requite my kindness? Ans. 2 years.

7. If a footman performs a journey in 3 days, when the days are 12 hours long, how many days will he require, of 9 hours long, to go the same journey? Ans. 4 days.

8. If 100 workmen complete a piece of work in 12 days, how many are sufficient to do it in 3 days? Ans. 400 men.

9. What number of men must be employed to finish in 5 days, what 15 men would be 20 days about? Ans. 60 men.

10. What length of board, that is six inches wide, will make a square foot? Ans. 2 feet.

11. If 30 bushels of wheat, at 50 cents per bushel, pay a debt, how many bushels at 75 cents will pay the same debt? Ans. 20 bushels.

12. If 8 boarders drink a barrel of beer in 12 days, how long would it last, if 4 more came among them? Ans. 8 days.

13. There is a cistern having a cock, which will empty it in 12 hours. I demand how many cocks of the same size there must be, to empty it in 15 minutes? Ans. 48 cocks.

14. If a piece of silk be 3 ells Flemish wide, and 4 ells Flemish long, I want to know how many yards it will take to line it, of stuff that is three-quarters of a yard wide? Ans. 9 yards.

15. How many yards of baize, three-quarters wide, will line a great coat, that has 12 yards of forest cloth in it, of half a yard wide? Ans. 8 yards.

16. There are 650 men stationed in a garrison, and their provisions are calculated to last them two months. How many men must leave the garrison, that the provisions may last what remain five months? Ans. 390 men.

17. What sum of money will gain as much interest in a month, as 400 dollars will gain in 12 months, at the same interest? Ans. 4800 dollars.

18. There was a certain building raised in 8 months, by 120 workmen; but, by a violent storm and inundation, the building was demolished. It is to be rebuilt in 2 months; I demand how many men must be employed about it?

Ans. 480 men.

19. What distance will 15 cwt. be carried, when 2 cwt. was carried 150 miles for 5 dollars? Ans. 20 miles.

20. What length of board, that is 9 inches broad, will measure 8 square feet? Ans. $10\frac{2}{3}$ feet.

Questions in Direct and Inverse Proportion.

1. An eagle and a cent just threescore yards did buy,
How many ells Flemish for fifteen dimes had I?

Ans. 11 ells Fl. 2 qrs. 3 na.+

2. A ship's company of 20 sailors is supposed to have provisions enough to last their voyage, at the rate of 12 oz. per day, when they pick up a crew in distress, of 4 persons. What will the daily allowance then be, and what is diminished off each former allowance?

Ans. 10 oz. daily allowance, and 2 oz. diminished.

3. A merchant bought a certain quantity of broadcloth, and baize, which cost 324 dollars. The quantity of broadcloth was 50 yards, at 3 dollars 60 cents a yard; and for every 5 yards of broadcloth, I had 9 yards of baize. I demand how many yards of baize he had, and what it cost him per yard?

Ans. 90 yds. and cost 1 dol. 60 cts. a yard.

4. I was looking on my watch one evening, when it thundered hard, and observed a flash of lightning, and in $8\frac{1}{2}$ sec. after, I heard the explosion. How far was it off where I then stood?* Ans. 1 mile 6 fur. 28 per. 1 yd. 2 ft.

5. What is the weight of a pea of a steelyard, which being suspended 40 inches from the centre of motion, will equipoise 300 lbs. suspended at the draught end, one-half of an inch? Ans. 3 lbs. 12 oz.

6. Suppose a large wheel in machinery to contain 60 cogs, and a smaller one in it 42 cogs, in how many revolutions of the larger wheel, will the lesser wheel gain 200 revolutions?

Ans. $466\frac{2}{3}$.

7. A seller of liquors retailed a pipe of wine, containing 120 gallons, with a measure that was found, on examination,

* Sir Isaac Newton allows sound uninterrupted to fly at the rate of 1142 feet per second. Whenever you see the flash of lightning, you never will get hurt by the explosion that follows, as the flash and explosion occur at the same instant.

to be deficient one gill of the standard measure of a gallon. How many of such gallons did the pipe of wine measure, and how much did he cheat his customers?

Ans. { Pipe of wine contained 123 gal. 3 qts.
Cheated his customers 3 gal. 3 qts.

8. Admit the hinder wheels of a wagon to be 12, and the forewheels 8 feet in circumference, how far will that wagon travel by the time the lesser wheels go round 500 times more than the large ones?

Ans. 2 miles 2 fur. 7 per. 1 yd. 1 ft. 6 inc.

9. In a wagon, whose large wheels are 12 feet, and the smaller ones 8 feet in circumference, how many more revolutions do the smaller wheels make than the larger ones, in going 2 miles 2 furlongs 7 perches 1 yard 1 foot 6 inches?

Ans. 500 revolutions.

10. When flour is sold at a dollar the 100 lbs. what is the first cost of a dozen of cakes, each weighing, one with another, 8 oz. allowing the bread to be in proportion to the flour, as 4 to 3; and what ought the selling price to be, allowing the flour to be to the bread, as 3 to 4?

Ans. 4½ cts. first cost, and 6 cts. the selling price.

11. How much land in length that is 20 rods wide, will make 4 acres? Ans. 32 rods.

THE DOUBLE RULE OF THREE DIRECT.

This Rule teaches at once to solve such questions as require two or more statings in simple proportion, whether direct or inverse. In this Rule, there are always five terms or numbers given to find a sixth. The three first terms or numbers are a supposition, the two last a demand.

To find whether in direct or inverse proportion.

Rule. In stating the question, place the terms of the supposition, so that the principal cause of loss, gain or action, possess the first place; that which signifies time, distance of place, &c. in the second place; and the remaining term in the third place. Place the terms of demand under those of the same kind in the supposition.

If the blank place, or term sought, fall under the third term, the proportion is *direct*. Then multiply the first and second terms together for a divisor, and the other three for a dividend. But if the blank fall under the first or second

term, the proportion is *inverse*. Then multiply the third and fourth terms together for a divisor, and the other three for a dividend, and the quotient will be the answer.

EXAMPLES.

1. If 7 men can make 36 rods of the turnpike road in 3 days, how many rods can 18 men make in 14 days?
Ans. 432 rods.

2. If 100 dollars in 12 months gain 6 dollars interest, how much will 500 dollars gain for the same time?
Ans. 30 dollars.

3. If 500 dollars in a year gained 30 dollars interest, how much will 100 dolls. gain for 12 months? Ans. 6 dolls.

4. If 20 bushels of corn are sufficient for 9 persons 5 months, how many bushels will serve 4 persons 12 months?
Ans. 21⅓ bushels.

5. If 8 men build 24 rods of brickwork in 6 days, how many men will build 18 rods in 3 days? Ans. 12 men.

6. If 400 dollars in 7 months gain 14 dollars, what is the rate per cent. per annum? Ans. 6.

7. What principal, at 6 per cent. per annum, being put to interest, will gain 14 dollars in 7 months?
Ans. 400 dollars.

8. If 100 dollars in a year gain 6 dollars, how long will 400 dollars be in gaining 14 dollars? Ans. 7 months.

9. If 12 bushels of oats suffice 20 horses for 20 days, how many bushels are sufficient for 60 horses 36 days?
Ans. 64⅘ bushels.

10. If 20 cwt. be carried 50 miles for 24 dollars, how much must be given for the carriage of 40 cwt. 100 miles?
Ans. 96 dollars.

11. If 14 dollars interest is gained by 400 dollars in 7 months, what sum will gain 6 dollars in 12 months, at the same rate? Ans. 100 dollars.

12. What is the interest of 947 dollars for 294 days, at 5 per cent. per annum? Ans. 38 dolls. 13 cts. 9 m.

THE DOUBLE RULE OF THREE INVERSE.

1. If 3 dolls. pay 8 men for 3 days work, how many days must 20 men work for 40 dollars? Ans. 16 days.

2. If 4 men are paid 20 dolls. for 3 days work, how many days must 16 men work for 300 dolls.? Ans. 11¼ days.

3. If 8 men can reap 80 acres of grain in 12 days, how many men can reap 100 acres in 5 days? Ans. 24 men.

4. If 8 men can reap 80 acres of grain in 12 days, how many days will it require 24 men to reap 100 acres?

Ans. 5 days.

5. If 5 dimes are paid for the carriage of 200 lbs. for 40 miles, how far may 20200 lbs. be carried for 60 dolls. 60 cts.?

Ans. 48 miles.

6. If 30 men can perform a piece of work in 20 days, how many men will accomplish another piece of work, four times as large, in a fourth of the time? Ans. 480 men.

7. If 8 men can build a wall 20 feet long, 6 feet high, and 4 feet thick, in 12 days, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick?

Ans. 80 days.

Promiscuous Questions.

1. If the carriage of 5 cwt. 3 qrs. 150 miles, cost 24 dollars 58 cents, what must be paid for the carriage of 7 cwt. 2 qrs. 25 lbs. 64 miles, at the same rate?

Ans. 14 dolls. 8 cts. 6 m.+

2. If the carriage of 8 hhds. of tobacco, each weighing 18 cwt. 2 qrs. 60 miles, cost 100 dollars, what must be paid for the carriage of 40 barrels of shad, each weighing 3 cwt. 3 qrs. 300 miles, at that rate?

Ans. 578 dollars 70 cents. +

3. If a footman travels 240 miles in 12 days, when the days are 14 hours long, in how many days will he perform a journey of 720 miles, when the days are 18 hours long?

Ans. 28 days.

4. If a pound of thread make 2 yards of cloth 6 quarters wide, how many pounds of thread will a piece of cloth 44 yards long, and 5 quarters wide, make?

Ans. $18\frac{1}{3}$ pounds.

5. In a pile of wood 300 feet long, 8 feet high, and 4 feet broad, how many cords are there, if a cord by law measures 8 feet in length, 4 feet wide, and 4 feet high?

Ans. 75 cords.

6. If 4 lbs. of woollen yarn make 12 yards of cloth 5 qrs. wide, how many pounds will it take to make 100 yds. long, and 4 qrs. wide? Ans. $26\frac{2}{3}$ lbs.

SECTION III.

VULGAR FRACTIONS.

Vulgar Fractions are broken numbers of an unit, or whole number, and are represented or shewn by two numbers, one placed above the other, with a line drawn between them, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, signifying a half, three-fourths, five-sixths of an unit, or whole number. The figure above the line is called the *numerator*, and that below the line is the *denominator*.

The denominator shews how many parts the integer is divided into, and the numerator shews how many of those parts you have. The denominator is a divisor in division, and the numerator is the remainder, after the quotient is sought.

Fractions are said to be in their lowest terms, when they are expressed by the least numbers possible, as $\frac{1}{2}$, $\frac{1}{4}$; but if $\frac{2}{4}$, $\frac{3}{6}$, they are not at their lowest, as 2 will divide the numerators and denominators of both, and bring them to be equal to $\frac{1}{2}$, $\frac{1}{3}$, which are their lowest terms possible. Vulgar Fractions have four names given them, by which they are known, viz. *proper, improper, compound and mixed.*

A proper fraction is when the numerator is less than the denominator, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{8}$, &c.

An improper fraction is when the numerator is equal to, or greater than the denominator, as $\frac{4}{4}$, $\frac{5}{3}$, $\frac{6}{6}$, $\frac{7}{3}$, $\frac{9}{2}$, $\frac{12}{11}$, &c.

A compound fraction is a fraction of a fraction, as $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{2}{3}$ of $\frac{3}{8}$, $\frac{4}{7}$ of $\frac{5}{8}$, &c.

A mixed fraction is a whole number, and a fraction annexed to it, as 6$\frac{1}{4}$ cts. 12$\frac{1}{2}$ cts. 18$\frac{3}{4}$ cts. &c.

REDUCTION OF FRACTIONS.

To reduce a fraction to its lowest terms.

Rule. Divide the greater term by the less, and that divisor by the remainder, till nothing be left; the last divisor is the common measure, by which divide both parts of the given fraction, and the quotient will be the answer. Or, divide the terms of the given fraction by any number which will divide them without a remainder, and the quotient again in like manner, and so on, till there is no number, greater than 1, which will divide them; then the least term is found.

EXAMPLES.

1. Reduce $\frac{56}{168}$ to its lowest terms. Ans. $\frac{1}{3}$.

56)168(3
168
—

Here 56 is the common measure, which divides both terms without a remainder. If ciphers be at the right hand of both terms, cut off an equal number from both.

2. Reduce $\frac{4}{8}$ to its lowest terms. Ans. $\frac{2}{3}$
3. Reduce $\frac{72}{144}$ to its lowest terms. $\frac{1}{2}$
4. Reduce $\frac{9}{12}$ to its lowest terms. $\frac{3}{4}$
5. Reduce $\frac{1440}{17280}$ to its lowest terms. $\frac{1}{12}$
6. Reduce $\frac{216000}{864000}$ to its lowest terms. $\frac{1}{4}$

To reduce an improper fraction to a whole or mixed number.

RULE. Divide the numerator by the denominator, and the quotient will be the whole number, and the remainder you set over the divisor, (if any there be,) for a numerator. The whole number found, and that fraction after division, will be the answer.

EXAMPLES.

1. Reduce $\frac{13}{4}$ to its proper terms. Ans. $3\frac{1}{4}$
2. Reduce $\frac{17}{5}$ to its proper terms. $3\frac{2}{5}$
3. Reduce $\frac{21}{7}$ to its proper terms. 3
4. Reduce $\frac{112}{28}$ to its proper terms. 4
5. Reduce $\frac{216}{8}$ to its proper terms. 27
6. Reduce $\frac{7480}{9}$ to its proper terms. $831\frac{1}{9}$
7. Reduce $\frac{67430}{11}$ to its proper terms. 6130

To reduce a mixed number to an improper fraction.

RULE. Multiply the whole number by the denominator of the fraction, and add the numerator to the product for a new numerator, under which place the given denominator.

EXAMPLES.

1. Reduce $3\frac{1}{2}$ to an improper fraction. Ans. $\frac{7}{2}$
2. Reduce $6\frac{1}{4}$ to an improper fraction. $\frac{25}{4}$
3. Reduce $9\frac{2}{3}$ to an improper fraction. $\frac{29}{3}$
4. Reduce $37\frac{1}{3}$ to an improper fraction. $\frac{75}{3}$
5. Reduce $62\frac{1}{4}$ to an improper fraction. $\frac{125}{4}$
6. Reduce $87\frac{1}{2}$ to an improper fraction. $\frac{175}{2}$
7. Reduce $146\frac{2}{3}$ to an improper fraction. $\frac{440}{3}$

To find the least common multiple, or denominator.

RULE. Divide the given denominators by any number that will divide two or more of them without a remainder, and set the quotients and undivided numbers underneath. Divide these quotients and undivided numbers by any number that will divide two or more of them, as before, and thus continue, till no two numbers are capable of being divided. Multiply the last quotients and divisor, or divisors, together, and the product will be the least common denominator required.

EXAMPLE.

What is the least common measure of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$?

2) 3 4 5 6 7 8 9
3) 3 2 5 3 7 4 9
2) 1 2 5 1 7 4 3
1 1 5 1 7 2 3
7
35
2
70
3
210 Product of all the undivided numbers.
2
420
3
1260
2
Ans. 2520 Least common measure.

To reduce several fractions to others, that shall have one common denominator, and still retain the same value.

RULE. Reduce the given fractions to their lowest terms; then multiply each numerator into all the denominators but its own, for a new numerator, and all the denominators into each other for a common denominator.

EXAMPLE.

Reduce $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{4}$, $\frac{3}{8}$, to a common denominator.

$3\times3\times4\times8=288$ common denominator.

$\frac{1}{3}$ numerator $1\times3\times4\times8=96$ the numerator for $\frac{1}{3}=\frac{96}{288}$
$\frac{2}{3}$ numerator $2\times4\times8\times3=192$ the numerator for $\frac{2}{3}=\frac{192}{288}$
$\frac{1}{4}$ numerator $1\times8\times3\times3=72$ the numerator for $\frac{1}{4}=\frac{72}{288}$
$\frac{3}{8}$ numerator $3\times4\times3\times3=108$ the numerator for $\frac{3}{8}=\frac{108}{288}$

A more precise way for finding a new numerator for a common denominator.

EXAMPLE.

1. Reduce $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{8}$, to a common denominator.

8)288 common denominator, as shewn before.

$\frac{1}{3}$ equal $\frac{96}{288}$
$\frac{2}{3}$ equal $\frac{192}{288}$
$\frac{1}{4}$ equal $\frac{72}{288}$
$\frac{3}{8}$ equal $\frac{108}{288}$

3)288 com. denom.
96 a new numerator for $\frac{1}{3}$
2
192 a new numerator for $\frac{2}{3}$

4)288 com. denom.
72
1
72 numerator for $\frac{1}{4}$

8)288 com. denom.
36
3
108 numerator for $\frac{3}{8}$

RULE for the above method. Divide each denominator separately into the common denominator, and multiply the quotient by the numerator belonging to the denominator you divide, and the product will be a new numerator for the common denominator.

2. What is the least number that can be divided by the nine digits separately, without a remainder?

Ans. 2520.

To reduce a compound fraction to a single one.

RULE. Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Such figures as are alike in the numerators and denominators may be cancelled.

1. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to a single fraction.

$$\left.\begin{array}{l} 2\times4\times5=40 \\ 3\times5\times6=90 \end{array}\right\} \frac{40}{90}=\frac{4}{9}$$ By the first part of the Rule.

By the cancelling method.

Numerators $2\times4\times5'=8$ $\frac{8}{18}=\frac{4}{9}$

Denominators $3\times5'\times6=18$

NOTE. The two 5's thus marked ' are omitted.

2. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{6}{7}$ to a single fraction.
Ans. $\frac{3}{14}$.

3. Reduce $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{5}$ of $\frac{4}{4}$ to a single fraction.
Ans. $\frac{3}{15}$.

4. Reduce $\frac{7}{8}$ of $\frac{6}{7}$ of $\frac{8}{9}$ of $\frac{11}{12}$ to a single fraction.
Ans. $\frac{11}{18}$.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

RULE. Make it a compound fraction, by comparing it with all the denominations between it and that to which it is to be reduced. Reduce this fraction to a single one.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of a cent to the fraction of a dollar.

$\frac{1}{2}\times\frac{1}{100}=\frac{1}{200}$ Ans. Or, $\frac{1}{2}$ of $\frac{1}{100}=\frac{1}{200}$ Ans.

2. Reduce $\frac{1}{4}$ of a cent to the fraction of a dime.
Ans. $\frac{1}{40}$.

3. Reduce $\frac{3}{4}$ of a dime to the fraction of a dollar.
Ans. $\frac{3}{40}$.

4. Reduce $\frac{3}{4}$ of a dollar to the fraction of an eagle.
Ans. $\frac{3}{40}$.

5. Reduce $\frac{4}{7}$ of a pennyweight to the fraction of a pound Troy.
Ans. $\frac{1}{350}$.

6. Reduce $\frac{4}{7}$ of a pound Avoirdupois to the fraction of a hundred weight.
Ans. $\frac{1}{175}$.

7. Reduce $\frac{1}{3}$ of a pint to the fraction of a gallon.
Ans. $\frac{1}{24}$.

8. Reduce $\frac{3}{4}$ of a quart to the fraction of a hogshead.
Ans. $\frac{1}{336}$.

9. Reduce $\frac{3}{4}$ of a minute to the fraction of a day.
Ans. $\frac{1}{1920}$.

10. Reduce $\frac{1}{2}$ of a day to the fraction of a month.
Ans. $\frac{1}{60}$.

11. Reduce $\frac{3}{4}$ of a day to the fraction of a year.
Ans. $\frac{3}{1460}$.

12. Reduce $\frac{1}{2}$ of a week to the fraction of a year.
Ans. $\frac{1}{104}$.

To reduce the fraction of one denomination to that of another, but less, retaining the same value.

RULE. Multiply the numerator by the parts of the denominator between it and that to which it is to be reduced, for a new numerator, and place it over the given denominator, which reduce to its lowest terms.

EXAMPLES.

1. Reduce $\frac{1}{200}$ of a dollar to the fraction of a cent.

$\frac{1}{200} \times \frac{100}{1} = \frac{100}{200} = \frac{1}{2}$ Ans.

2. Reduce $\frac{1}{40}$ of a dime to the fraction of a cent.
Ans. $\frac{2}{5}$.

3. Reduce $\frac{3}{40}$ of a dollar to the fraction of a dime.
Ans. $\frac{3}{4}$.

4. Reduce $\frac{3}{40}$ of an eagle to the fraction of a dollar.
Ans. $\frac{3}{4}$.

5. Reduce $\frac{1}{300}$ of a pound Troy to the fraction of a penny-weight.
Ans. $\frac{4}{5}$.

6. Reduce $\frac{1}{196}$ of a cwt. to the fraction of a pound Avoirdupois.
Ans. $\frac{4}{7}$.

7. Reduce $\frac{1}{16}$ of a gallon to the fraction of a pint.
Ans. $\frac{1}{2}$.

8. Reduce $\frac{1}{336}$ of a hogshead to the fraction of a quart.
Ans. $\frac{3}{4}$.

9. Reduce $\frac{1}{1920}$ of a day to the fraction of a minute.
Ans. $\frac{3}{4}$.

10. Reduce $\frac{1}{60}$ of a month to the fraction of a day.
Ans. $\frac{1}{2}$.

11. Reduce $\frac{3}{1460}$ of a year to the fraction of a day.
Ans. $\frac{1}{4}$.

12. Reduce $\frac{1}{104}$ of a year to the fraction of a week.
Ans.

To find the value of a fraction in the known parts of the integer.

RULE. Multiply the numerator by the known parts of the integer, and divide by the denominator.

EXAMPLES.

1. What is the value of $\frac{3}{4}$ of a dollar?

One dollar = 100 cts. $\times$ 3=300$\div$4=75 cts. Ans.

2. What is the value of $\frac{1}{2}$ of an eagle? Ans. 5 dolls.
3. What is the value of $\frac{1}{4}$ of a dime?
Ans. 2 cents 5 mills.
4. What is the value of $\frac{1}{2}$ of a cent? Ans. 5 mills.
5. What is the value of $\frac{1}{2}$ of a ton? Ans. 10 cwt.
6. What is the value of $\frac{2}{3}$ of a hogshead?
Ans. 42 gallons.
7. What is the value of $\frac{3}{4}$ of a pound Avoirdupois?
Ans. 12 oz.
8. What is the value of $\frac{2}{3}$ of a day? Ans. 16 hours.
9. What is the value of $\frac{3}{5}$ of a minute? Ans. 40 sec.

To reduce any given quantity to the fraction of a greater denomination of the same kind.

RULE. Reduce the given quantity to the lowest denomination mentioned, for a new numerator, under which set the integral part, (reduced,) to the same name, for a denominator.

EXAMPLES.

1. Reduce 75 cents to the fraction of a dollar.

Set 75 cents over 100 cents, and divide both by 25, and the quotients are $\frac{3}{4}$. $25)\frac{75}{100} = \frac{3}{4}$ Ans.

2. Reduce 5 dollars to the fraction of an eagle.
Ans. $\frac{1}{2}$.
3. Reduce 2 cents 5 mills to the fraction of a dime.
Ans. $\frac{1}{4}$.
4. Reduce 5 mills to the value of a cent. Ans. $\frac{1}{2}$.
5. Reduce 10 cwt. to the fraction of a ton. Ans. $\frac{1}{2}$.
6. Reduce 42 gallons to the fraction of a hogshead.
Ans. $\frac{2}{3}$.
7. Reduce 12 oz. to the fraction of a pound Avoirdupois.
Ans. $\frac{3}{4}$.
8. Reduce 16 hours to the fraction of a day. Ans. $\frac{2}{3}$.
9. Reduce 40 sec. to the value of a minute. Ans. $\frac{2}{3}$.

To reduce a vulgar fraction to a decimal of the same value.

RULE. Add ciphers to the numerator of the given fraction, and divide by the denominator.

EXAMPLES.

1. Reduce $\frac{1}{2}$ to a decimal of the same value. Ans. ,5
2. Reduce $\frac{1}{4}$ to a decimal of the same value. ,25
3. Reduce $\frac{3}{4}$ to a decimal of the same value. ,75
4. Reduce $\frac{5}{6}$ to a decimal of the same value. ,8333+
5. Reduce $\frac{7}{8}$ to a decimal of the same value. ,875

ADDITION OF VULGAR FRACTIONS.

To add fractions that have the same common denominators.

RULE. Add all the numerators together, and divide the sum by the common denominator, and the quotient will be a whole number; and place the remainder, (if any there be,) over the divisor, for a denominator; and place that fraction on the right hand of the whole number.

EXAMPLES.

1. Add $\frac{2}{12}$, $\frac{3}{12}$, $\frac{5}{12}$, and $\frac{7}{12}$, together.

Numerators, $2 + 3 + 5 + 7 = 17 \div 12 = 1$, and 5 remains, which I place over 12, the divisor, making $\frac{5}{12}$, which I set down thus, $1\frac{5}{12}$, answer.

2. Add $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{6}{8}$, and $\frac{7}{8}$, together. Ans. $2\frac{3}{4}$.
3. Add $\frac{1}{15}$, $\frac{2}{15}$, $\frac{3}{15}$, $\frac{4}{15}$, $\frac{7}{15}$, and $\frac{9}{15}$, together. $1\frac{1}{15}$.
4. Add $\frac{11}{24}$, $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$, and $\frac{23}{24}$, together. $3\frac{1}{4}$.
5. Add $\frac{9}{36}$, $\frac{11}{36}$, $\frac{14}{36}$, $\frac{17}{36}$, and $\frac{35}{36}$, together. $2\frac{7}{18}$.

To add fractions of different denominators.

RULE. Reduce the given fractions to a common denominator, and proceed as in the last.

1. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$, together.

24 com. denom.

3	8 × 2	= 16
4	6 × 3	= 18
6	4 × 5	= 20
8	3 × 7	= 21
12	2 × 11	= 22

$97 \div 24 = 4\frac{1}{24}$ Ans.

2. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{9}$, and $\frac{13}{14}$, together. Ans. $3\frac{409}{630}$.
3. Add $\frac{7}{8}$, $\frac{4}{15}$, $\frac{6}{15}$, $\frac{11}{15}$, and $\frac{14}{15}$, together. $3\frac{941}{1560}$.

To add mixed numbers of different denominators.

RULE. Add the fractions as in the last case, and the integers as in whole numbers.

EXAMPLES.

1. Add $3\frac{1}{2}$, $7\frac{2}{3}$, $9\frac{5}{12}$, and $1\frac{5}{6}$, together. Ans. $24\frac{1}{4}$.
2. Add $7\frac{1}{4}$, $3\frac{2}{5}$, $8\frac{3}{16}$, and $21\frac{11}{12}$, together. $40\frac{47}{80}$.
3. Add $10\frac{5}{7}$, $14\frac{3}{14}$, $21\frac{4}{21}$, and $30\frac{2}{42}$, together. $76\frac{4}{7}$.

To add fractions of different denominations together.

RULE. Reduce them to their proper quantities, as in reduction, and add as before.

EXAMPLES.

1. Add $\frac{1}{2}$ of an eagle, and $\frac{3}{4}$ of a dollar together.

$\frac{1}{2}$ of an eagle = $ 2 50 cts.
$\frac{3}{4}$ of a dollar = 0 75
Ans. $ 3 25

$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 75$ cts.
$\frac{1}{2} \times \frac{10}{1} = \frac{10}{4} = 250$ cts.

2. Add $\frac{4}{7}$ of a dollar, $\frac{8}{9}$ of a dime, and $\frac{3}{4}$ of a cent together. Ans. $84\frac{3}{4}$ cents.
3. Add $\frac{1}{2}$ of a dollar, $\frac{1}{8}$ of a dollar, and $\frac{4}{7}$ of a cent together. Ans. $63\frac{3}{10}$ cents.
4. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile together. Ans. 1540 yds. 2 ft. 9 inc.
5. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. Ans. 2 days $14\frac{1}{2}$ hours.
6. Add $\frac{3}{4}$ of a mile, and $\frac{7}{10}$ of a furlong together. Ans. 6 furlongs 28 poles.
7. Add $\frac{4}{7}$ of a ton to $\frac{9}{10}$ of a hundred weight. Ans. 12 cwt. 1 qr. 8 lbs. $12\frac{1}{10}$ oz.
8. Add $\frac{1}{2}$ of a month, $\frac{2}{3}$ of a day, $\frac{3}{4}$ of an hour, $\frac{1}{5}$ of a minute, and $\frac{4}{7}$ of a second together. Ans. 15 days 16 hrs. 45 min. $20\frac{4}{7}$ sec.
9. Add $\frac{3}{4}$ of a yard, and $\frac{2}{3}$ of an ell Flemish together. Ans. 1 yd. 1 qr.
10. Add $\frac{1}{2}$ of a hogshead, $\frac{1}{4}$ of a gallon, and $\frac{3}{4}$ of a quart together. Ans. 31 gal. 3 qts. 1 pt. 2 gills.
11. Add $\frac{1}{2}$ of a yard, $\frac{2}{3}$ of a foot, and $\frac{3}{4}$ of an inch together. Ans. 2 ft. $2\frac{1}{4}$ inc.

SUBTRACTION OF VULGAR FRACTIONS.

RULE. Prepare the fractions as in addition, and take the lower numerator from the upper, and place the difference over the common denominator.

Mixed numbers.

When the lower numerator is greater than the upper numerator, you may, without reducing them to improper fractions, subtract the numerator of the lower fraction from the common denominator, and to that difference add the upper numerator, carrying 1 to the units' place of the lower whole number.

Also, a fraction may be subtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator; then take 1 from the whole number.

Of a common denominator.

From	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{5}{8}$	$\frac{9}{10}$	$\frac{11}{12}$	$\frac{13}{14}$	$\frac{17}{18}$	$\frac{23}{24}$	$\frac{71}{83}$	$\frac{199}{300}$
Take	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{5}{7}$	$\frac{3}{8}$	$\frac{3}{10}$	$\frac{5}{12}$	$\frac{11}{14}$	$\frac{7}{18}$	$\frac{17}{24}$	$\frac{29}{83}$	$\frac{113}{300}$
Ans.	$\frac{1}{2}$	$\frac{2}{6}$	$\frac{1}{7}$	$\frac{1}{4}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{7}$	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{3}{83}$	$\frac{43}{150}$

Of different denominators.

From	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{9}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{11}{12}$	$\frac{14}{15}$
Take	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{14}$	$\frac{3}{18}$	$\frac{3}{15}$	$\frac{7}{12}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{2}{3}$
Ans.	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{9}{14}$	$\frac{7}{18}$	$\frac{3}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{4}{15}$

Of mixed numbers, having a common denominator.

From	$214\frac{3}{5}$	$376\frac{7}{10}$	$200\frac{31}{63}$	$398\frac{3}{4}$	$940\frac{19}{24}$
Take	$192\frac{5}{5}$	$260\frac{4}{10}$	$198\frac{42}{63}$	$276\frac{1}{4}$	$800\frac{23}{24}$
Ans.	$21\frac{3}{5}$	$115\frac{1}{2}$	$1\frac{32}{63}$	$114\frac{1}{2}$	$139\frac{5}{6}$

Of divers denominations, and quality.

1. From 7 weeks take $9\frac{7}{10}$ days.
   Ans. 5 weeks 4 days 7 hrs. 12 min.
2. From $5\frac{1}{2}$ dollars take $37\frac{1}{2}$ cents.
   Ans. 5 dollars $12\frac{1}{2}$ cents.

3. From $3\frac{3}{5}$ eagles take $19\frac{1}{2}$ dollars.
Ans. 13 dolls. $12\frac{1}{2}$ cts.
4. From 3 tons take $2\frac{1}{2}$ cwt.
Ans. 2 tons 17 cwt. 2 qrs.
5. From $6\frac{1}{4}$ miles take $5\frac{1}{2}$ yards.
Ans. 6 miles 5 fur. 39 poles.
6. From $3\frac{1}{4}$ yards take 1 rod. Ans. 3 yds. 9 inc.
7. From $\frac{1}{2}$ of a hogshead take $1\frac{1}{4}$ gallons.
Ans. 30 gallons 1 quart.
8. From $\frac{1}{4}$ of a hundred weight take $\frac{3}{4}$ of a pound.
Ans. 2 qrs. 27 lbs. 4 oz.
9. From $\frac{1}{3}$ of a day take $\frac{1}{3}$ of an hour.
Ans. 20 hours 45 minutes.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE. Reduce whole and mixed numbers to improper fractions, mixed fractions to simple ones, and those of different integers to the same; then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; and if it be an improper fraction, bring it to a mixed one.

EXAMPLES.

1. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.	Ans. $\frac{8}{15}$
2. Multiply $\frac{1}{3}$ by $\frac{3}{7}$.	$\frac{3}{8}$
3. Multiply $\frac{5}{6}$ by $\frac{8}{9}$.	$\frac{20}{27}$
4. Multiply $\frac{1}{4}$ by $\frac{1}{2}$.	$\frac{1}{8}$
5. Multiply 2 by $2\frac{1}{2}$.	5
6. Multiply $2\frac{1}{2}$ by $\frac{1}{2}$.	$1\frac{1}{4}$
7. Multiply $2\frac{1}{2}$ by $2\frac{1}{2}$.	$6\frac{1}{4}$
8. Multiply $8\frac{1}{4}$ by $\frac{3}{4}$.	$6\frac{3}{16}$
9. Multiply $21\frac{1}{4}$ by $6\frac{3}{5}$.	$135\frac{1}{2}\frac{5}{0}$

DIVISION OF VULGAR FRACTIONS.

RULE. Prepare the fractions as before; then invert the divisor, and proceed exactly as in multiplication.

EXAMPLES.

1. Divide $\frac{1}{2}$ by $\frac{1}{4}$.

Thus, $\frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$ Ans.

2. Divide $\frac{8}{15}$ by $\frac{4}{5}$. Ans. $\frac{2}{3}$.
3. Divide $\frac{3}{8}$ by $\frac{3}{7}$. $\frac{7}{8}$
4. Divide $\frac{20}{27}$ by $\frac{8}{9}$. $\frac{5}{6}$
5. Divide $\frac{1}{3}$ by $\frac{1}{2}$. $\frac{2}{3}$
6. Divide 5 by $2\frac{1}{2}$. 2
7. Divide $1\frac{1}{4}$ by $\frac{1}{2}$. $2\frac{1}{2}$
8. Divide $6\frac{3}{16}$ by $\frac{3}{4}$. $8\frac{1}{4}$
9. Divide $6\frac{1}{4}$ by $2\frac{1}{2}$. $2\frac{1}{2}$
10. Divide $135\frac{1}{32}$ by $6\frac{3}{8}$. $21\frac{1}{4}$

RULE OF THREE IN VULGAR FRACTIONS.

RULE. Prepare the fractions as before taught; then state your question, and invert the first term in the proportion. Then multiply the three numerators together for a new numerator, and all the denominators together for a new denominator, and that will be the answer in the same name with the second term.

EXAMPLES.

1. If $\frac{2}{5}$ of a yard of muslin cost $37\frac{1}{2}$ cents, how much will $42\frac{1}{3}$ yards cost, at that rate? Ans. 23 dolls. $81\frac{1}{4}$ cts.
2. Bought $\frac{3}{5}$ of a ship for $940\frac{3}{4}$ dollars; what is the value of the whole ship, at that rate?
Ans. 1567 dollars $91\frac{3}{4}$ cents.
3. Sold $8\frac{2}{3}$ tons of iron for $152\frac{8}{15}$ dollars; how much will $1\frac{1}{2}$ pounds cost, at that rate? Ans. 8 cents.
4. At $\frac{3}{4}$ of a dollar a yard, how much will 80 yards come to? Ans. 60 dollars.
5. If I paid $1\frac{1}{2}$ cents for $1\frac{1}{2}$ herrings, how many can I get for 11 cents? Ans. 11 herrings.
6. Bought 3 hhds. of tobacco, each weighing $7\frac{3}{4}$ cwt. at the rate of $7\frac{1}{2}$ cents a pound. How much does the tobacco come to? Ans. $195\frac{3}{16}$ dollars.
7. If $\frac{1}{2}$ of a ship be worth 1000 dollars, what part of her may be purchased for 800 dollars? Ans. $\frac{4}{5}$.

RULE OF THREE INVERSE IN VULGAR FRACTIONS.

RULE. Prepare the fractions, and state the question for inverse proportion; then invert the third term, which is your divisor, and multiply the three numerators together for a new numerator, and the denominators together for a new denominator, and that will be the answer.

1. How much cloth, that is $\frac{3}{8}$ of a yard wide, will line $5\frac{1}{2}$ yards, that is $1\frac{3}{4}$ yards wide?

As $1\frac{3}{4}$ yds. : $5\frac{1}{2}$ yds. :: $\frac{3}{8}$ yd. Thus, $\frac{7}{4}\times\frac{11}{2}\times\frac{3}{8}=\frac{385}{24}=16\frac{1}{24}$ Ans.

2. If a man performs a journey in $3\frac{1}{2}$ days, when the days are $12\frac{1}{3}$ hours long, in how many days will he do it, when the days are $9\frac{1}{4}$ hours long? Ans. $4\frac{50}{117}$ days.

3. When the days are $9\frac{1}{4}$ hours long, a man performs a journey in $4\frac{50}{117}$ days. In how many days will he do it, when the days are $12\frac{1}{3}$ hours long? Ans. $3\frac{1}{2}$ days.

4. If a man performs a journey in $3\frac{1}{2}$ days, when the days are $12\frac{1}{3}$ hours long, how long must the days be, when he will perform the same journey in $4\frac{50}{117}$ days?

Ans. $9\frac{1}{4}$ hours.

5. When a man performs a journey in $4\frac{50}{117}$ days, when the days are $9\frac{1}{4}$ hours long, how long must the days be, when he will perform the same journey in $3\frac{1}{2}$ days?

Ans. $12\frac{1}{3}$ hours.

6. If the $\frac{1}{3}$ of 6 be 3, how much will be the $\frac{1}{4}$ of 20? *(Direct Proportion.)* Ans. $7\frac{1}{2}$.

SECTION IV.

PRACTICE.

Rule. Practice is a contraction of the Rule of Three Direct, when the first number is 1, or a unit, and is a concise method of calculating most questions that occur in business, where sterling money is reckoned; and for which reason it was, and is called Practice. But in this country, this Rule is almost useless; for which reason, I will not give many questions in it.

Multiply the whole numbers of the given numbers together, and take aliquot parts for the rest.

TABLES.

2 cents	is	$\frac{1}{50}$	of a dollar.	20 cents	is	$\frac{1}{5}$	of a dollar.
4	"	$\frac{1}{25}$	"	25	"	$\frac{1}{4}$	"
5	"	$\frac{1}{20}$	"	40	"	$\frac{2}{5}$	"
$6\frac{1}{4}$	"	$\frac{1}{16}$	"	50	"	$\frac{1}{2}$	"
10	"	$\frac{1}{10}$	"	80	"	$\frac{4}{5}$	"
$12\frac{1}{2}$	"	$\frac{1}{8}$	"	90	"	$\frac{9}{10}$	"

WEIGHTS.

1 pound is	$\frac{1}{112}$	of a cwt.	21 lbs. is	$\frac{3}{16}$	of a cwt.
7 "	$\frac{1}{16}$	"	28 "	$\frac{1}{4}$	"
14 "	$\frac{1}{8}$	"	56 "	$\frac{1}{2}$	"
16 "	$\frac{1}{7}$	"	84 "	$\frac{3}{4}$	"

MEASURES.

1 gill is	$\frac{1}{32}$	of a gallon.	4 poles is	$\frac{1}{10}$	of a furlong.
1 pint	$\frac{1}{8}$	"	5 poles	$\frac{1}{8}$	"
1 quart	$\frac{1}{4}$	"	8 poles	$\frac{1}{5}$	"
1 nail	$\frac{1}{16}$	of a yard.	10 poles	$\frac{1}{4}$	"
2 nails	$\frac{1}{8}$	"	20 poles	$\frac{1}{2}$	"
1 quarter	$\frac{1}{4}$	"	2 pints	$\frac{1}{64}$	of a bushel.
2 quarters	$\frac{1}{2}$	"	1 quart	$\frac{1}{32}$	"
1 furlong	$\frac{1}{8}$	of a mile.	2 quarts	$\frac{1}{16}$	"
2 furlongs	$\frac{1}{4}$	"	1 peck	$\frac{1}{8}$	"
4 furlongs	$\frac{1}{2}$	"	2 pecks	$\frac{1}{4}$	"

1. What is the amount of 146 yards of cloth, at 1 dollar 50 cents per yard?

	146	yards.
	1 50	
	146	dollars, at 1 dollar per yard.
50 cts. is $\frac{1}{2}$	73	dollars, at 50 cents per yard.
Ans.	219	dollars.

			$	cts.		$	cts.
2.	264 yards	at	1	12	Ans.	295	68
3.	417	"	1	60		250	20
4.	326½	"	1	87		610	55½
5.	514	"	3	20		1644	80
6.	370	"	3	50		1295	00
7.	492¾	"	2	62½		1293	46⅞
8.	312	"	2	31¼		721	50
9.	654 lbs.	sugar		6¼		40	87½
10.	800½	butter		8		64	04
11.	362¾	steel		8		29	02
12.	327	pork		4½		14	71½

When the price is the fractional part of a dollar, or a cent.

RULE. Multiply the given number by the numerator, and divide the product by the denominator, and the quotient will be the answer.

EXAMPLES.

1. What will 240 yards of cloth come to, at $\frac{3}{4}$ of a dollar per yard?

240
3

720÷4=180 dollars. Ans.

2. What will 3 cwt. 2 qrs. 14 lbs. of sugar come to, at $\frac{1}{16}$ of a dollar per pound? Ans. 25 dolls. 37½ cts.

3. What is the price of 4 cwt. and 16 lbs. of butter, at $\frac{4}{5}$ of a dollar a pound? Ans. 58 dollars.

4. What is the cost of 18 cwt. 3 qrs. of hay, at $\frac{1}{5}$ of a dollar per cwt.? Ans. 3 dolls. 75 cts.

5. What is the amount of 23 cwt. 1 qr. 21 lbs. of iron, at $4\frac{1}{3}$ dollars per cwt.? Ans. 101 dolls. 56¼ cts.

6. Bought three barrels of salt, each containing 5 bushels 2 pecks, at $\frac{2}{3}$ of a dollar per bushel. What does the salt amount to? Ans. 10 dolls. 31¼ cts.

7. Bought four boxes of tobacco pipes, the first two boxes contained each 7½ gross, and the other two contained each 14 score 1 dozen and 4, at the rate of $\frac{3}{4}$ of a cent for every two tobacco pipes. What is the whole amount?

Ans. 10 dolls. 32 cts.

8. Sold 17 cwt. 3 qrs. 19 lbs. of sugar, at 10 dolls. 94 cts. per 112 lbs. How much is the amount?

Ans. 196 dollars 4 cents.

9. Bought six hampers of cheeses, each hamper contained eight cheeses, and each cheese weighed 12½ lbs. at 6¾ dolls. the 112 lbs. How much is the amount of the cheese?

Ans. $36\frac{3}{7}$ dollars.

10. What is the amount of 324 barrels of flour, each weighing 186 lbs. at $2\frac{3}{8}$ dollars per 112 lbs.?

Ans. $1277\frac{40}{84}\frac{3}{2}$ dollars.

11. How much is the amount of 19 tons 19 cwt. 3 qrs. 27½ lbs. at 9 eagles 9 dollars 9 dimes 9 cents 9½ mills the ton? Ans. 199 eag. 9 dolls. 9 dim. 6 cts. $7\frac{40}{84}\frac{84}{560}$ mills.

TARE AND TRET.

Tare and Tret are practical Rules for deducting certain allowances, which are made by merchants, in buying and selling goods, &c. by weight; and in which are noticed the following.

Gross weight is the whole weight of goods, together with the cask, box, bag, or any other thing, which contains such goods.

Tare is an allowance made to the buyer for the weight of the cask, box, bag, &c. which contains the goods bought, and is either so much per cask, &c. or so much per hundred weight, or so much in the whole weight.

Tret is an allowance of 4 pounds, on every 104 pounds, for dust, and for retailing in smalls.

Cloff is an allowance of 2 lbs. upon every 3 cwt.

Suttle is what remains after one or two allowances have been deducted.

RULE. Add the gross weight into one sum, and the tare into another; then subtract the total tare from the whole or total gross, and the remainder will be the neat or suttle weight.

EXAMPLES.

1. Bought six barrels of flour, each weighing 226 pounds gross, tare 14 lbs. per barrel. How much is the neat weight, and how much must I pay for the flour, at 4 dollars 6¼ cents per 100 pounds?

Ans. { Neat weight 1272 lbs.
Cost 51 dolls. 67 cts. 5 m.

2. How much is the neat weight of 16 hhds. of tobacco, each weighing 5 cwt. 1 qr. 19 lbs. gross, tare 100 lbs. per hogshead, and what does it come to, at 8 dollars 96 cents per cwt.?

Ans. { Neat weight 72 cwt. 1 qr. 20 lbs.
Amount 647 dollars 36 cents.

3. In four hogsheads of sugar, two of which weighed each 9 cwt. and the other two each 11 cwt. 3 qrs. 12 lbs. gross, tare 75 lbs. per hogshead, how much is the amount, at 6¼ cents per pound, and what is the neat weight?

Ans. { Amount $273 25 cts.
Neat weight 39 cwt. 0 qr. 4 lbs.

4. What is the neat weight of four barrels of sugar, weight as follows, and the cost, at 7 dolls. 84 cts. per 112 lbs.?

	cwt.	*qrs.*	*lbs.*		*lbs.*
No. 1.	4	1	0	Tare	37
2.	3	3	12		28
3.	4	0	26		33
4.	3	3	21		34

Ans. { Neat weight 15 cwt. 0 qr. 11 lbs.
Cost 118 dollars 87 cents.

5. What is the amount of four tierces of rice, gross 8 cwt. 2 qrs. 14 lbs. tare 21 lbs. per tierce, at 4 dollars the 100 lbs. and how much is the neat weight?

Ans. { Amount 79 dolls. 52 cts.
Neat weight 17 cwt. 3 qrs.

6. Bought four chests of tea, each weighing gross 1 cwt. 12 lbs. tare 21 lbs. per chest. What is the neat weight, and what does the tea come to, at $\frac{3}{4}$ of a dollar per pound?

Ans. { Neat weight 3 cwt. 2 qrs. 20 lbs.
Cost 309 dollars.

7. Sold 214 firkins of butter, 100 of which weighed each 2 qrs. 16 lbs. and the remainder weighed 2 tons 17 cwt. gross; tare on the first hundred firkins, 14 lbs. each, and on the remainder, 12 lbs. each. How much is the neat weight, and what does the butter come to, at 8 dollars 96 cents the hundred weight?

Ans. { Neat weight 4 tons 16 cwt. 2 qrs. 8 lbs.
Amount 865 dollars 28 cents.

8. At 21 cents per pound, what will 6 hhds. of coffee come to, and what is the neat weight, being marked and numbered as follows:

	cwt.	*qrs.*	*lbs.*		*lbs.*
No. 1.	7	1	10	Tare	96
2.	8	2	25		100
3.	7	1	24		93
4.	6	3	27		86
5.	7	0	24		93
6.	8	1	6		100

Ans. { Cost 963 dollars 48 cents.
Neat weight 2 tons 3 qrs. 24 lbs.

9. Bought 4 hhds. of brandy; each hogshead had leaked; but the merchant of whom I bought, to save the trouble of gauging them, allows a deduction of 15 per cent. on the whole. How many gallons am I to pay for, and what is the amount, at $1\frac{3}{4}$ dollars the gallon?

Ans. { $214\frac{1}{2}$ gallons.
Amount 294 dolls. 52 cts. 5 m.

NOTE. I count the hogshead at 63 gallons.

When tret is allowed with the tare.

RULE. Find the tret first, and subtract it from the gross, and the remainder is called suttle. Then find the tret by dividing the suttle by 26, as 4 lbs. is the $\frac{4}{26}$ part of 104 lbs. which subtract from the suttle, and the remainder will be the neat weight.

1. In a hogshead of tobacco, weighing gross 10 cwt. 1 qr. 12 lbs. tare 14 lbs. per 112 lbs. tret 4 lbs. per 104 lbs. how much neat weight? Ans. 8 cwt. 2 qrs. 24 lbs.

2. How much is the gross weight of the above hogshead of tobacco, at the above tret and tare? *

Ans. 10 cwt. 1 qr. 12 lbs.

3. In 25 barrels of Nantz currants, weighing gross 18 cwt. 3 qrs. tare 12 lbs. per 112 lbs. tret 4 lbs. per 104 lbs. how many pounds neat, and how much is the amount, at 25 cts. per pound?

Ans. { 1803+ lbs. neat.*
Amount $450 75 cts.

4. What is the neat weight of two hogsheads of sugar, each weighing gross 7 cwt. 1 qr. 7 lbs. tare 16 lbs. per 112 lbs. tret 4 lbs. per 104 lbs. and what is the amount, at 7 dollars 84 cents per 112 lbs.?

Ans. { Neat weight 12 cwt. 6 lbs.
Amount 94 dollars 50 cents.

5. What is the gross weight of a hogshead of sugar, weighing neat 11 cwt. 1 qr. tare 14 lbs. per cwt.?

Ans. 12 cwt. 3 qrs. 12 lbs.

6. In 70 bales of silk, each 317 lbs. gross, tare per bale 16 lbs. how many pounds neat? Ans. 21070 lbs.

7. In 12 hhds. of sugar, weighing gross 177 cwt. 22 lbs. tare 9 lbs. per cwt. tret 4 lbs. on every 104 lbs. how many hundred weight neat? Ans. 156 cwt. 2 qrs. 22 lbs.

8. In a hogshead of lard, weight gross 8 cwt. 3 qrs. 20 lbs. tare 38 lbs. tret 4 lbs. per 104 lbs. what is the neat weight, and what does it come to, at 5 cents a pound?

Ans. { Neat weight 8 cwt. 1 qr. 1 lb.
Cost 46 dollars 25 cents.

9. Bought 4 hhds. of sugar, weighing gross 41 cwt. 3 qrs. 13 lbs. tare 321 lbs. tret 4 lbs. on every 104 lbs. cloff 2 lbs. on every 3 cwt. How much neat, and what is the amount, at 7 dollars per cwt.?

Ans. { Neat weight 4175 lbs.
Amount $260 93¾ cts.

BARTER

Is the exchanging of one commodity for another, and directs merchants and others how to make an exchange, without loss to either party.

* I did not work for the fraction.

RULE. Find the value of the commodity, whose quantity is given; then find what quantity of the other, at the proposed rate, can be bought for the same money, and that will be the answer.

EXAMPLES.

1. What quantity of flax, at 6¼ cents a pound, must I give for 12 lbs. of coffee, at 25 cents a pound?

Here you must find how much the coffee comes to, by saying,

lb. cts. lbs.
If 1 : 25 :: 12 : 3 dollars.

Now, you have the amount of the coffee; then you must find how many pounds of flax, at 6¼ cents a pound, will come to $3, by saying,

cts. lb. dolls.
If 6¼ : 1 :: 3 : 48 lbs. Ans.

2. How much wheat, at 1 dol. 25 cts. a bushel, may be given for 50 bushels of rye, at 70 cents per bushel?
Ans. 28 bushels.

3. How many barrels of herrings, at 2 dollars per barrel, must I give in barter for 140 barrels of flour, at 4 dollars 70 cents the barrel? Ans. 329 barrels.

4. A buys of B 4 barrels of whiskey, each containing 32 gallons, at 25 cents per gallon, for which I am to give him rye, at 37½ cents a bushel. How many bushels of rye am I to give B for said whiskey? Ans. 85⅓ bushels.

5. C barters 4 hhds. of tobacco, weighing 1 ton 18 cwt. 2 qrs. 14 lbs. neat, at 9 cts. per pound, with D, for 96 dolls. in cash, and the balance in corn, at 31¼ cts. a bushel. How many bushels of corn is D to receive?
Ans. $938\frac{86}{125}$ bushels.

6. A bought of B the hull of a schooner, of 70 tons burthen, at 16 dollars per ton, and paid him in cash 500 dolls. 3 hhds. of molasses, containing each 116⅔ gallons, at 6 dimes 4 cents per gallon, and the balance he is to pay in gin, at 74 cents a gallon. How many gallons of gin is B to receive? Ans. $535\frac{5}{37}$ gallons.

7. B has tea, which stands him a dollar per pound; but, bartering with C for coffee, he will charge 1 dollar 25 cents a pound. C's coffee stands him 20 cts. a pound. At what rate per pound must C charge his coffee, to make the barter equal? Ans. 25 cents.

8. F bartered with G 8¾ cwt. of tobacco, at 12 cents per pound, for which G gave him 18 cwt. of flour. How was the flour rated a pound? Ans. 5⅓ cents nearly.

9. A has salt at 5 dollars per barrel, which he barters to B, at a profit of 25 per cent. for rum, which cost 1 dollar 25 cents a gallon. At what rate must C sell his rum, to make an even barter? Ans. 1 dol. 56¼ cts.

10. C barters with D 36 lbs. 8 oz. of cloves, at 1 dollar 50 cents per pound, for flax, at 6¼ cents a pound. How much flax is C to receive? Ans. 7cwt. 3 qrs. 8 lbs.

11. A and B barters: A has muslin that cost him 20 cts. a yard, and he puts it at 25 cents; B has calico, which stands him 30 cents a yard. At what price must he put it, to gain 10 per cent. more than A? Ans. 41¼ cents.

12. C has 300 yards of Irish linen, which stands him a dollar per yard: this he barters with D, at 1 dollar 5 cents a yard, for rum, which stands D 1 dollar 9 cents a gallon. How many gallons of rum is C to receive in barter, and at what price, that their gains may be equal?

Ans. $262\frac{94}{153}$ gallons, at $1 $14\frac{9}{20}$ cts. a gallon.

PROFIT AND LOSS

Is a Rule by which merchants and traders discover their profit or loss, in buying and selling goods. It also instructs them how to rise or fall in the price of their goods, so as to gain or lose so much per cent. &c.

EXAMPLES.

1. Bought a barrel of whiskey, containing 32 gallons, for 8 dollars, and afterwards I retailed it out, at 37½ cents per gallon. How much did I gain on the barrel?

gal. cts. galls.
If 1 : 37½ :: 32 : 12 dollars.

Sold for 12 dollars.
Bought for 8

Gained 4 dollars.

2. Bought 100 peaches, at 2 for a cent, and 100 apples, at 3 for a cent; and afterwards I sold them all, at 5 for 2 cents. Whether do I gain or lose by this trade?

Ans. I lose 3⅓ cents.

3. Bought 3½ cwt. of iron for 15 dollars 68 cents, and retailed it out at 6¼ cents a pound. How much is my gain on the whole, and how much per pound?

Ans. { Whole gain $8 82 cts.
Gain per pound 2¼ cents.

4. Bought a hogshead of molasses, containing 119 gallons, at 52 cents per gallon; paid for cartage 1 dollar 25 cents; and by accident 9 gallons leaked out. At what rate per gallon must I sell the remainder, to gain 13 dollars on the whole? Ans. 69 cents 2 mills.+

5. If a pound of coffee is sold for 28 cents, which costs 25 cents, what is the profit on 8 bags, each weighing 1 cwt. 14 lbs.? Ans. 13 dollars 86 cents.

6. A merchant bought 4 hogsheads of tobacco, weighing 28 cwt. 1 qr. 14 lbs. for 8 dollars the hundred pounds; paid sundry expenses, amounting to 15 dollars 75 cents; and sold the same for 450 dollars. What is his profit on the whole, and what per pound?

Ans. { Profit on the whole $180 1 ct.
Profit per pound 5 cts. $6\frac{10}{13}\frac{2}{9}\frac{1}{9}$ m.

7. Bought a hamper, containing a gross bottles of porter, for 5 dollars 76 cents, and sold them singly at 6¼ cents a bottle. How much is my gain per bottle?

Ans. 2¼ cents.

To know what is gained or lost per cent.

RULE. Find what the gain or loss is by subtraction; then say, as the price it cost is to the gain or loss, so is 100 dollars to the gain or loss per cent.

EXAMPLES.

8. If I buy whiskey at the rate of 25 cents per gallon, and sell it again at 32 cents a gallon, what do I gain per cent.?

Sold for 32 cents.
Bought for 25

Gained 7 cents in laying out 25 cts. Then say,

If 25 cts. gained 7 cts. what will 100 dolls. gain? Ans. $28.

9. If I buy flour at 3 dollars 44 cents per barrel, and sell it again for 4 dollars 30 cents a barrel, what do I gain per cent.? Ans. 25 per cent.

10. When 20 per cent. loss is made on coffee, which was sold at 25 cents per pound, what was the first cost? Ans. 31¼ per cent.

11. If I make 12½ cents profit on a dollar, how much is that per cent.? Ans. 12½ per cent.

12. Bought a piece of Irish linen, containing 52 yards, at 50 cents per yard, and sold it at 12½ per cent. loss. At how much did I sell it per yard? Ans. 43¾ cents.

13. A merchant bought 40 dozen of wool hats, at 90 cts. apiece. At what rate must he sell them, to gain 25 per cent.? Ans. 1 dollar 12½ cents.

14. Bought a pipe of wine, containing 120 gallons, at 1 dollar 15 cents per gallon. How many gallons of water must be put in it, to gain 10 dollars, by selling it at a dollar per gallon? Ans. 28 gallons.

15. A merchant bought 2 cwt. of tea for 224 dollars, and on account of some of it being damaged, was obliged to sell it at 87½ cents a pound. What is the loss on the whole, and how much per cent. and per pound?

Ans. { Loss on the whole is 28 dolls.
Loss per pound is 12½ cts.
Loss per cent. is 12½.

16. If by selling 400 yards of muslin for 200 dollars, there is 20 per cent. profit, what did the muslin cost per yard? Ans. 41⅔ cents.

17. Bought 164 yards of broadcloth, at 3 dolls. per yard, ready money, and sold the same again for 618 dollars, on 6 months credit. What did I gain by the whole, allowing 6 per cent. discount per year?* Ans. 108 dollars.

18. If I buy cloth at 4 dollars 16 cents per yard, on 8 months credit, and sell it again at 3 dollars 90 cents per yard, ready money, whether do I gain or lose per cent. and how much, allowing discount at 6 per cent. per annum? Ans. I lose 2½ per cent.

19. Which is the best bargain—in purchasing fish, at 3 dollars 40 cents per barrel, and 4 months credit, or to pay 3 dollars 33⅓ cents in cash down? Ans. They are alike.

To know how a commodity must be sold, to gain or lose so much per cent.

Rule. As 100 is to the purchase price, so is 100 with the profit added, or the loss subtracted, to the selling price.

20. If I buy flour at 5 dollars per barrel, how must I sell it to gain 25 per cent.? Ans. 6 dollars 25 cents.

21. If I buy cloth at 2 dolls. 25 cts. per yard, how must I sell it to gain 30 per cent.? Ans. 2 dolls. 92½ cts.

22. If tea costs 54 cents a pound, how must it be sold to lose 12½ per cent.? Ans. 47 cts. 2½ m.

* When goods are bought or sold on credit, you must calculate, (by discount,) the present worth of their price, in order to find the true gain or loss.

23. Bought broadcloth at 3 dolls. 50 cts. a yard, which not proving so good as I expected, I am willing to lose 15 per cent. to get it off hands. How must I sell it per yard?
Ans. 2 dollars 97½ cents.

24. Bought 90 gallons of wine, at 1 dollar 20 cents per gallon; but by accident 10 gallons leaked out. At what rate must I sell the remainder per gallon, to gain upon the whole prime cost 12½ per cent.?
Ans. 1 dollar 51 cents $8\frac{7}{10}$ mills.

25. A sells a quantity of wheat at a dollar per bushel, and gains 20 per cent. Some time after, he sold of the same, to the amount of 50 dollars, and gained 50 per cent. How many bushels were in the last parcel, and at what rate did he sell it per bushel?
Ans. 40 bushels, at 1 dol. 25 cts. per bushel.

26. A merchant is about purchasing 2500 gallons of whiskey, which he can have at 48 cents per gallon, in ready money, or 50 cents, with two months credit. Which is more advantageous to him, either to buy on credit, or to borrow the money, at 8 per cent. per annum, to pay the cash price? Ans. He will save $34 by paying the cash.

When there is gained or lost per cent. to know what the commodity cost.

RULE. As 100 dollars, with the gain per cent. added, or loss per cent. subtracted, is to the price, so is 100 to the prime or first cost.

27. If a yard of broadcloth be sold for 3 dollars 50 cents, and there is gained 70 dollars 50 cents per cent. what was the prime cost of that yard?
Ans. 2 dollars 5 cents 2 mills.+

28. If by selling broadcloth at 3 dollars 25 cents a yard, I lose at the rate of 20 per cent. what was the first cost of said cloth per yard? Ans. 4 dolls. 6 cts. 2½ m.

29. If 50 pounds of chocolate be sold at 25 cts. a pound, and I gain 9 per cent. what did the whole cost me?
Ans. 11 dollars 46 cents 6 mills.+

30. Bought 5 cwt. of sugar, and sold it at 12 cents per pound, by which I gained at the rate of 25½ per cent. What did the sugar cost me per cwt.?
Ans. 10 dollars 70 cents 9 mills.+

If by wares, sold at a given rate, there is so much per gained or lost, to know what would be gained or lost cent. if sold at another rate.

RULE. As the first price is to 100 dollars, with the p per cent. added, or the loss per cent. subtracted, so is other price to the gain or loss per cent. at the other p But if your answer be 100 dolls. the excess is your gain cent.; but if it be less than 100 dollars, the deficiency is loss per cent.

31. If I sell cloth at a dollar per yard, and thereby 15 per cent. what shall I gain per cent. if I sell it at 1 20 cts.? Ans. 38 per cen

32. If I retail tea at 1 dol. 50 cts. per pound, and the gain 25 per cent. what will I gain or lose, if I sell it at 1 10 cts. per pound? Ans. I lose $8\frac{1}{3}$ per cen

33. If by selling 1 cwt. of tobacco for 8 dollars, I los per cent. what will I gain or lose, if I sell 6 cwt. for 54 lars? Ans. I lose 1 per cen

34. Sold my horse for 120 dollars, and by so doing, 15 per cent. whereas I ought, in trading, to have cleare per cent. How much did I sell my horse under his value? Ans. 49 dolls. 41 cts. 1 m.

100—15=85 dollars.

	dolls.	*dolls.*	*dolls.*	*dolls.*	*cts.*	*m.*	
As	85 :	120 : :	100 :	141	17	$6\frac{8}{17}$	first or prime

	dolls.	*dolls.*	*dolls.*	*cts.*	*m.*		*dolls.*	*cts.*
As	100 :	120 : :	141	17	$6\frac{8}{17}$	:	169	41
						Sold for	120	
						Ans.	$49	41

FELLOWSHIP

Is a Rule by which the accounts of several merchant other trading persons, in partnership, are so regulated, each may have his share of the gain or loss, in proporti his share of the joint stock. The estates of bankrupt divided proportionably among their creditors by this rul

SINGLE FELLOWSHIP

Is when the several shares of stock are continued in t an equal term of time.

RULE. Add the several stocks together, and say, as that sum is to the whole gain or loss, so is each man's stock to his particular share of gain or loss.

PROOF. Add all the particular shares of the gain or loss together, and if it be equal to the whole gain or loss, the question is right.

EXAMPLES.

1. A and B buy flour to the amount of 480 dollars, of which A paid 320 dollars, B 160 dollars; and they gained thereby 128 dollars. How much is each man's share of the profit?

A put in 320 dollars.
B 160

				dolls.	cts.	
As 480	: 128 ::	320	:	85	33⅓	A's gain.
As 480	: 128 ::	160	:	42	66⅔	B's gain.

2. D and E were partners in a parcel of merchandise, in the purchase of which D laid out 12 dollars, and E 28 dollars. The commodity being sold, they find their gain to be 5 dollars. What part of it must each man have?

Ans. D must have $1 50 cts. and E $3 50 cts.

3. A, B and C, trading together, gained 480 dolls. which is to be shared among them, according to each man's stock. A put in 560 dollars, B 1200 dollars, and C 640 dollars. What is each man's share?

Ans. A's share $112, B's $240, C's $128.

4. M and N have gained 1200 dollars, whereof M is to have 10 per cent. more than N. What is the share of each?

Ans. M's share $628⅔, N's $571⅓.

5. Three merchants, B, C and D, shipped off 108 horses to Candia, of which B owned 48, C 36, and D 24; but in distress at sea, the sailors were obliged to throw 45 overboard. I demand how much of the loss each owner must sustain? Ans. B 20, C 15, and D 10.

6. Three merchants, trading in tobacco, lost to the sum of 3200 dollars. Now, if D's stock was 4800 dollars, E's 19200 dollars, and F's 8000 dollars, what sum did each man lose? Ans. D lost $480, E $1920, and F $800.

7. Four merchants traded with a stock of 3200 dollars, and gained in two years twice as much, and 160 dolls. more. A's stock was 560 dollars, B's 1040 dollars, and F's 1200 dollars. I demand G's stock, and each man's gain?

Ans. { G's stock is 400 dollars.
{ A gained $1148, B $2132, F $2460, G $820.

8. A bankrupt is indebted to B 844 dollars, to C 1200 dollars, and to D 1564 dolls. and his whole estate is worth only 2702 dollars, which he gives up to his creditors. How much must each have, in proportion to his debt?

Ans. { B must have $632 5+ cts. / C must have 898 67+ / D must have 1171 27

9. Three men rent a farm, containing 400 acres, for 600 dollars, for a certain time, of which B paid 120 dollars, C 240, and D the remainder; and they agree that the farm shall be divided in proportion to the sums paid by each. I demand each man's share?

Ans. B's share 80 acres, C and D each 160 acres.

10. Three men have to share a legacy of 1500 dollars, of which A is to have ⅓, B ¼, and C the remainder; but B relinquishes ¼ to A, and C leaving it to be divided between them, according to their shares in the whole. It is required to know how much of the legacy A and C receive respectively? Ans. A's part $1000, and C's $1500.

11. A father left an estate of 6000 dollars to three sons, in such a manner, that for every two dollars A got, B should have three, and C five. How much did each son receive?

Ans. A $1200, B $1800, and C $3000.

12. Two merchants have gained 1800 dollars, of which B is to have three times as much as C. How much is each to receive? Ans. B receives $1350, and C $450.

13. A, B and C, traded in company; A put in 140 dollars, B 250 dollars, and C put in 130 yards of cloth, at cash price. They gained 230 dollars, of which C took 100 dollars, for his share of the gain. How did C value his cloth per yard, in common stock, and what was A and B's part of the gain?

Ans. { C valued his cloth at $2 80/13 cts. a yard. / A gained 46 dolls. 66 cts. 6 m.+ / B gained 83 dolls. 33 cts. 3 m.+

COMPOUND FELLOWSHIP,

Or Fellowship with time, is occasioned by several partners being continued in trade an unequal term of time.

Rule. Multiply each man's time by his stock in trade, and add the several products together, and say, as the sum of the several products is to the whole gain or loss, so is each man's product to his particular share of the gain or loss.

EXAMPLES.

1. A, B and C, hold a pasture in common, for which they pay 76 dollars. A put in 8 horses for 6 weeks, B 12 for 8 weeks, and C 12 for 12 weeks. What must each pay of the rent?

A put in	8	× 6	=	48
B put in	12	× 8	=	96
C put in	12	× 12	=	144
				288

	$			$		
As 288 :	76	::	48	:	12	66⅔ cts. A pays.
			96	:	25	33⅓ B pays.
			144	:	38	00 C pays.

2. B and C traded in company; B put in $800 dollars for 5 months, and C 3140 dollars for 6 months, and they gained 1103 dollars 66⅔ cents. What is each man's share of the profit? Ans. B's $554 16⅔ cts. and C's $549 50 cts.

3. B put in a stock of 1800 dollars, and C, at the end of six months, put in a stock that entitled him, at the end of the year, to an equal share of the gain. How much did C put in? Ans. 3600 dollars.

4. Three persons had received 665 dollars interest. B put in 4000 dollars for a year, C 3000 for 15 months, and D 5000 for 8 months. How much is each man's part of the interest? Ans. B's $240, C's $225, and D's $200.

5. Three merchants enter into partnership for 16 months. B put into stock at first 600 dollars, and at the end of six months 200 dollars more. C put in at first 1200 dollars; but at the end of 12 months, was obliged to take out 600 dollars. D put in at first 1000 dollars, and at the end of 12 months, put in 800 dollars more. With this stock they gained 2800 dolls. What is each man's share of the gain?

Ans. B's 560\frac{60}{119}$, C's 811\frac{91}{119}$, D's 927\frac{87}{119}$.

6. B and C got into partnership; B put in, first January, a thousand dollars; and C put in, first of May following, a sum which entitled him to an equal share of the gain with B, at the year's end. How much did he put in?

Ans. 1500 dollars.

7. Two merchants hired a sleigh, in the winter, to go to Pittsburgh from Washington, (G. C. Ohio,) and back, for 40 dollars, the distance 100 miles between both places, with conditions, that they might take in two other passengers. Now, when at Wheeling, they took in C, and when at Pittsburgh, on their return, they took in D to Wheeling, where

they also left C. Now, the distance from here, (Washington,) to Wheeling, is 40 miles. It is required to divide the sleigh-ride among them, in proportion to the miles they travelled?

Ans. { The two merchants each paid $13 $79\frac{9}{19}$ cts.
C paid $8 $27\frac{17}{19}$ cts.
D paid 4 $13\frac{2}{19}$

EQUATION OF PAYMENTS

Is finding the supposed equated time, to pay at once several debts, due at different periods of time, so that no loss shall be sustained by either party.

RULE. Multiply each payment by its time, and divide the sum of the several products by the whole debt, and the quotient will be the equated time.

The correct Rule is, to find the present worth of each particular sum; then find on what time these present worths will be increased to the total of the particular sums, payable at the particular times to come, and that is the true equated time for the payment of the whole debt.

EXAMPLES.

1. B owes C 880 dollars, to be paid as follows, viz. 100 dollars in six months, 120 in eight months, and 160 in twelve months. What is the equated time for the payment of the whole debt? Ans. 9 months 4+ days.

100	×	6	= 600
120	×	8	= 960
160	×	12	= 1920
880			)3480(9 months 4+ days.

2. B is indebted to C 480 dollars, whereof ⅓ is to be paid in three months, ¼ in six months, and the remainder in nine months. Required the equated time for the payment of the whole debt? Ans. 5 months 7½ days.

3. M owes N 1000 dollars, whereof 200 dollars is to be paid at present, 400 dollars at five months, and the rest at fifteen months; but they agree to make one equated payment of the whole debt. Required the time?

Ans. 8 months.

4. C owes D 1400 dollars, to be paid in three months; but D being in want of money, C pays him 1000 dollars at the expiration of two months. How long then, after three months, may he in justice defer the payment of the rest?

Ans. $2\frac{1}{2}$ months.

NOTE. This question is solved by inverse proportion; for C paid 1000 dollars one month before the debt was due: therefore D allows C to keep the remaining 400 dollars so long, as will requite him for the 1000 dollars he received one month before due. Thus,

Debt 1400 dollars, due in 3 months.
Paid 1000. dollars in 2 months.

As 1000 : 1 :: 400 : $2\frac{1}{2}$ months.

5. A merchant has due to him a certain sum of money, to be paid $\frac{1}{6}$ at two months, $\frac{1}{3}$ at three months, and the rest at six months. What is the equated time for the payment of the debt?

Ans. $4\frac{1}{2}$ months.

POSITION

Is a Rule which, by false or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts, Single and Double.

SINGLE POSITION

Is when one number is required, the properties of which are given in the question.

RULE. Take any number, and set it down, and proceed as the question directs; then say, as the result of the operation is to the given sum in the question, so is the supposed number to the true one required.

PROOF. Substitute the answer in the place of the supposed number, and proceed as before.

EXAMPLES.

1. A man, driving his hogs to market, was asked by another how many he had. He answered he could not tell, but said, that the $\frac{1}{3}$ and $\frac{1}{4}$ of them, added together, made 70. How many had he?

Ans. 120.

Suppose he had 60

$\frac{1}{3}$ of 60 equal 20
$\frac{1}{4}$ of 60 equal 15

As 35 : 70 : : 60 : 120 Ans.

2. A schoolmaster being asked how many scholars he had, said, if I had as many more as I have now, half as many, one-third, and one-fourth as many, I should then have 148. How many scholars had he? Ans. 48 scholars.

3. What sum is that, the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{7}$ of which, will make 308? Ans. 240.

4. A man, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had 60 dollars left; how many had he at first? Ans. $144.

5. A gentleman a chaise did buy,
A horse and harness too;
They cost two hundred and forty dollars,
Upon my word 'tis true.
The harness came to half of the horse,
The horse twice of the chaise.
If you find the price of them,
Take them, if you please.

Ans. Harness $60, chaise $60, horse $120.

6. A person left his son a sum of money, for which he is to receive interest, when he comes of age, at 6 per cent. per annum, simple interest. Now, at the expiration of 5 years, he received, for principal and interest, 644 dollars 80 cents. What was the sum left his son? Ans. $496.

7. Three persons discoursing together concerning their ages, says B, I am 20 years of age; says C, I am as old as B, and half D; and says D, I am as old as you both. I demand the age of each person?

Ans. B 20, C 60, and D 80 years.

8. A vessel has three cocks, B, C and D. B can fill it in one hour, C in two hours, and D in four hours. In what time will they all fill it together?

Ans. In 34 min. 17$\frac{1}{7}$ sec.

Suppose B 40 parts.
C equal to 20
D equal to 10

min.
As 70 : 60 :: 40 : 34 min. 17$\frac{1}{7}$ sec. Ans.

Or, B 1
C $\frac{1}{2}$
D $\frac{1}{4}$

min.
As 1$\frac{3}{4}$: 60 :: 1 : 34 min. 17$\frac{1}{7}$ sec.

DOUBLE POSITION

Teaches to solve questions, by making two supposed numbers.

RULE. Take any two convenient numbers, and proceed with each, according to the conditions of the question; then find how much the results are different, than the results in the question.

Multiply the first supposed number by the last error, and the last supposed number by the first error. If the errors are alike, divide the difference of their products by the difference of the errors, and the quotient will be the answer. If the errors are unlike, divide the sum of their products by the sum of the errors, and the quotient will be the answer. The errors are alike, when they are both too great, or both too small; and unlike, when one is too great, and the other too small.

EXAMPLES.

1. A, B and C, would divide 400 dollars between them, so as B may have 12 dollars more than A, and C 16 dollars more than B. I demand how much each man must have?

Suppose A had	$100	Suppose A had	$104
B + 12 =	112	B	116
C + 16 =	128	C	132
	340		352
Should be	400	Should be	400
Error too little by	60	Error too little by	48

1st sup. 100 × 104 2nd sup.

1st error 60 × 48 last error.

From 6240 − 4800

Take 4800

1440 ÷ 12 = 120 A's

120 + 12 = 132 B's

132 + 16 = 148 C's

400 Proof.

1st error 60

2nd error 48

Diff. 12

2. A man, lying at the point of death, said he had in a certain chest 400 dollars, which he bequeathed to his three sons, after this manner: the first must have a certain sum,

the second twice as much, wanting 32 dollars, and the third must have three times as much as the first, wanting 60 dollars. I demand how much each man gets?

Ans. The first $82, second $132, third $186.

3. A, B and C, built a house, which cost 400 dollars, of which A paid a certain sum, B paid 40 dollars more than A, and C paid as much as A and B. I demand each man's share of the building?

Ans. A paid 80 dolls. B 120, and C 200.

4. A laboring man hired with a farmer for 60 days, upon this condition, that for every day he worked, he should receive 80 cents, and for every day he was idle, he should pay 40 cents for his boarding. At the expiration of the time, he received 30 dolls. How many days did he work, and how many was he idle?

Ans. Wrought 45 days, and idle 15.

5. Divide 15 into two such parts, so that when the greater is multiplied by 4, and the lesser by 16, the products will be equal. Ans. 12 and 3.

6. Two persons, A and B, have both the same income. A saves $\frac{1}{5}$ of his yearly; but B, by spending 150 dollars per annum more than A, at the end of 8 years finds himself 400 dollars in debt. What is their income, and what does each spend per year?

Ans. { Their income is 500 dollars a year.
A spends 400 dollars, and B 550.

Suppose their income $400
A saves one-fifth of his 80

A spends 320
B spends 150

B spends in one year 470 × 8 = 3760 dollars in 8 years
B's income 400 × 8 = 3200

560
400 in debt.

160 error.

Suppose their income $450
A saves one-fifth 90

A spends 360
B spends 150

510 × 8 = 4080
450 × 8 = 3600

480
400 in debt.

80 error.

1st sup. 400 450 2nd sup.

160 80

72000
32000

160 — 80 = 80)40000

Ans. $500 ÷ 5 = 100, which he saves.

500—100 = 400, which he spends.
B spends 150

550 × 8 = 4400 dollars, spent in 8 years.
Income 500 × 8 = 4000

In debt $400 at the expiration of 8 years.

7. A certain man having driven his swine to market, viz. hogs, sows and pigs, received for them all 200 dollars, being paid for every hog 3 dollars 60 cents, for every sow 3 dolls. 20 cts. and for every pig 40 cents. There were as many hogs as sows, and for each sow there were three pigs. I demand how many there were of each sort?

Ans. 25 hogs, 25 sows, and 75 pigs.

8. What number is that, which being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 18 more, will be doubled? Ans. 72.

9. A person being asked the hour of the day, answered, that the time past, since noon, was equal to $\frac{2}{13}$ of the time remaining until midnight. Required the time?

Ans. 36 minutes past 1.

10. A gentleman has two horses, and a cart worth forty dollars, which cart, if put to the first horse, will make his value double to that of the second; but if the cart be put to the second horse, it will make his value triple to that of the first horse. What is the value of each horse?

Ans. First horse $24, second $32.

Suppose the first horse worth $18
The cart 40

Double the second 58

Value second horse $ 29
Cart 40

69
Triple first horse 54

Error 15

Suppose the first horse worth $20
Cart 40

Double the second 60

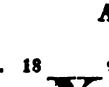
Value second horse $ 30
Cart 40

70
Triple first horse 60

Error 10

Errors 15 — 10 = 5 difference.

1st sup. 18 20 2nd sup.



15 10

300 180 $300 - 180 = 120 \div 5 = \$24$ the price of the first horse.

Then $\$24$ the price of the first horse.
40 the price of the cart.

$64 \div 2 = 32$ dollars, the price of the second horse.

PROOF.

1st horse $24 + 40 = 64 \div 2 = 32 =$ the 2nd horse.
2nd horse $32 + 40 = 72 \div 3 = 24 =$ the 1st horse.

11. There is a fish, whose head is nine inches long, and his tail is as long as his head, and half his body, and his body is as long as the head and tail. What is the whole length of the fish? Ans. 72 inches.

12. A surly old fellow being demanded the ages of his four children answered, you may go and look; but if you must know, my first son was born just one year after I was married to his mother, who, after his birth, lived five years, and then died in childbed, with my second son. Four years after that, I married again, and within two years, had my third and fourth sons at a birth, the sum of whose two ages is now equal to that of the eldest. I demand their several ages?

Ans. The first son was 22 years old, the second 17, the third 11, and the fourth 11.

A useful Rule.

Any question, however intricate, and though incapable of being answered by common Position, may be solved by the following rule: Proceed as in Double Position, only let your supposed numbers be as near the truth as possible, and then, by the rule of Proportion, say, as the difference of the two errors, if alike, or their sum, if unlike, is to the difference of the two positions, so is the least error to the difference between its position, and the first approximate value of the number sought. Repeat the operation with this approximate value, and the position which gave the least error, for a further approximation, and so on, to any degree of accuracy required.

EXAMPLE.

13. Let it be required to divide 100 into two such parts, that the product of the whole, into one of the parts, may be equal to the square of the other part?

1st. Suppose 38 and 62. Error in this, 44—
2nd. Suppose 38.2 and 61.8. Error in this, .76+ } 38.2—38=.2

Errors 44+.76=44.76; then say, as 44.76 : .2 : : .76 to .034, which is the difference between its position, (38.2,) and the first approximate value of the number sought. Therefore it will be 38.2 — .034 = 38.166, the approximate value of the least part, and very near the truth. By repeating the operation, you will come to the exact number, which, taken from 100, will leave the greatest number.

14. Two merchants put equal sums in trade: A gained a sum equal to one-fourth of his stock, and B lost 225 dolls. Then A's gain and stock was double that of B's. What sum did each put in? Ans. 600 dollars.

15. A gentleman had two silver cups, of unequal weight, having one cover to both, 5 oz. Now if the cover is put on the less cup, it will make it double the weight of the greater cup; but if put on the greater cup, it will be three times the weight of the less cup. What is the weight of each cup?
Ans. The less 3 oz. and greater 4 oz.

16. A saves the one-third of his wages; but B, who has the same salary, by spending twice as much as A, sinks 52 dollars a year. What is their yearly salary?
Ans. 156 dollars each.

Suppose each had	$ 120	Suppose each had	$ 108
A saves one-third	40	A saves one-third	36
A spends	80	A spends	72
	2		2
B spends	160	B spends	144
B's salary	120	B's salary	108
	40		36
Should be	52	Should be	52
Error too little	12	Error too little	16

1st sup. 120 108 2nd sup.

12 16

1296 1920
1296

Errors 16 — 12 = 4)624

Ans. 156 dollars, salary of each.

SECTION V.

INTEREST

Is of two kinds, Simple and Compound. Simple Interest is the sum paid by a borrower to a lender for the use of money lent, and is generally at a certain rate per cent. per annum, and is fixed by law, in several of the states, at 6 dollars for the use of 100 dollars for one year.

Principal is the sum lent, bearing interest.

Rate is the sum per cent. agreed on.

Amount is the principal and interest added together.

To find the interest of any sum for a year.

RULE. Multiply the principal by the rate per cent. and divide the product by 100, and the quotient will be the answer. Or say,

As $100 : $6 :: $200 to $12 for a year.

EXAMPLES.

1. Lent a friend 150 dollars for a year; how much is he to give me for principal and interest, at the year's end, at 6 per cent.? Ans. 159 dollars.

As 100 : 6 :: 150 : $9 interest.
150 principal.

Ans. $159

2. How much is the interest of 125 dollars for one year, at 6 per cent. per annum? Ans. 7 dolls. 50 cts.

3. What is the interest of 400 dollars for two years, at 6 per cent. per annum? Ans. 48 dollars.

4. How much is the interest of 300 dollars for a year and a half, at 6 per cent. per annum? Ans. 27 dolls.

5. What is the amount of 500 dollars for two years and a half, at 6 per cent. per annum? Ans. 575 dolls.

6. How much is the amount of 1000 dollars for a year and a quarter, at 6 per cent. per annum? Ans. 1075 dollars.

7. What is the interest of 240 dollars for one year, at 5 per cent. per annum? Ans. 12 dollars.

8. Required the interest of 314 dollars for one year, at 7 per cent. per annum? Ans. 21 dolls. 98 cts.

9. What is the amount of 560 dollars, due in one year, at 8 per cent. per annum? Ans. 604 dolls. 80 cts.

10. Required the interest of 760 dollars for one year, at 7 per cent. per annum? Ans. 53 dolls. 20 cts.

When the time is more than one year.

RULE. Find the interest for one year, and multiply that by the number of years, and the product will be the answer.

11. What sum will discharge a debt of 250 dollars, due two years, at 6 per cent. per annum? Ans. 280 dolls.

12. Required the interest of 375 dollars for five years, at 7 per cent. per annum? Ans. 131 dolls. 25 cts.

When the principal is dollars and cents, or dollars, cents and mills, and the time years only.

RULE. Multiply the given sum by the rate per cent. and divide the product by 100, and the quotient will be the answer in the same name with the lowest denomination in the principal, for one year only; but if for two or more years, multiply the interest found for one year by the number of years, and that product will be the answer.

13. What is the interest of 250 dollars 50 cents for one year, at 6 per cent. per annum? Ans. 15 dolls. 3 cts.

14. Required the amount of the above sum for four years, at 7 per cent. per annum? Ans. 320 dolls. 64 cts.

15. What is the amount of a note for 800 dolls. 75 cts. due five years, at 8 per cent. per annum?

Ans 1121 dollars 5 cents.

16. What sum will discharge a bond of 630 dolls. 50 cts. on which there is five years interest due, at 8 per cent. per annum? Ans. 882 dollars 70 cents.

17. How much is the interest of 325 dolls. for three years, at 6 per cent. per annum? Ans. 58 dolls. 50 cts.

18. What is the interest of 178 dollars 50 cents for two years, at 6 per cent. per annum?

Ans. 20 dollars 82 cents.

19. What is the interest of 279 dollars 87 cents for two years and a half, at 7 per cent. per annum?

Ans. 48 dollars $97\frac{7}{10}+$ cts.

20. What is the interest of one dollar for one year, at 6 per cent. per annum? Ans. 6 cents.

21. What sum will discharge a bond for 500 dollars 50 cents, due four years and a half, at 7 per cent. per annum?

Ans. 658 dollars 15 cents $7+$ mills.

When the principal is dollars, cents, &c. the time years and months, or months only, and when the rate per cent. contains a fraction, as ½, ⅓, ¼, &c.

RULE. Multiply by half the number of months in the given time, when the rate is at 6 per cent. per annum; but if the given rate be more or less than 6 per cent. multiply the given number of months by the rate, and divide the product by 12, and the quotient will be the rate for the time. The principal multiplied by this rate will give the interest required; and when the fraction occurs, you must multiply the principal by the fraction, as well as the whole number, and add it in.

22. How much is the interest of 187 dollars 25 cents for 1 year 4 months, at 6 per cent. per annum?
Ans. 14 dollars 98 cents.

23. What is the interest of 341 dollars 52 cents for 7½ months, at 6 per cent. per annum?
Ans. 12 dollars 80 cents 7 mills.

24. What is the interest of 400 dollars 50 cents for 2 years 8 months, at 7½ per cent. per annum?
Ans. 80 dollars 9 cents 9 mills.

25. What is the interest of 342 dollars for 11 months, at 6 per cent. per annum? Ans. 18 dolls. 81 cts.

26. What is the interest of 95 dollars for two months, at 6 per cent. per annum? Ans. 95 cents.

27. How much is the amount of 250 dollars 80 cents for 18 months, at 8¾ per cent. per annum?
Ans. 283 dollars 71 cents 7 mills.

28. What is the interest of 694 dollars 84 cents for nine months, at 10 per cent. per annum?
Ans. 52 dollars 11 cents 3 mills.

29. How much is the amount of 985 dollars for 5 years 8 months, at 6 per cent. per annum?
Ans. 1319 dollars 90 cents.

30. What is the amount of 1260 dollars for four months, at 6½ per cent. per annum? Ans. 1287 dolls. 30 cts.

31. What is the amount of 240 dollars for 15 months, at 7½ per cent. per annum? Ans. 262 dollars 50 cents.

When the principal is dollars, cents, &c. and the time is months and days, or days only.

RULE. Find the interest for the given months by the last rule, and take aliquot parts for the days; or multiply the given sum by the number of days, (when the rate is 6 per cent.) and divide the product by 60, and the quotient will

be the answer required. In taking aliquot parts for days, the calculation is made at 30 days to a month. By this method of taking aliquot parts for days, it is not exactly sufficient, where there is a large sum to be calculated; but on small sums the difference is very small. The correct way is to find the interest for the months, and then, by the Rule of Three, say, as 365 days is to the interest for one year, so is the number of days to the interest they will bring, which, added to the interest found for months, will give the answer.

32. What is the interest of 194 dollars for 4 months 12 days, at 6 per cent. per annum?

Ans. 4 dollars 26 cents 8 mills.

33. Required the interest of 640 dollars 60 cents for 100 days, at 6 per cent. per annum?

Ans. 10 dollars 53+ cents.

34. What is the amount of 481 dollars 75 cents for 25 days, at 7 per cent. per annum?

Ans. 484 dollars 5 cents 9+ mills.

35. What is the amount of 564 dollars for 44 days, at 6 per cent. per annum? Ans. 568 dolls. 7cts. 9+ m.

36. What is the interest of 87 dolls. 56 cts. for 72 days, at 6 per cent. per annum? Ans. 1 dol. 3 cts. 6 m.

37. What is the amount of 8400 dollars for 20 days, at 5 per cent. per annum? Ans. 8423 dollars.

38. How much is the amount of 298 dolls. 59 cts. from the 19th May, 1826, to the 11th August, 1827, at 8 per cent. per annum? Ans. 327 dollars 98 cents 4 mills.

39. How much interest is due on a note, passed the 14th of June, 1827, for 196 dollars, at 5 per cent. per annum, on the 14th of December following? Ans. 4 dolls. 90 cts.

40. Required the amount of 73 dollars 41 cents for 27 days, at 6 per cent. per annum? Ans. 73 dolls. 74 cts.

41. How much is the interest of 35 dollars for 256 days, at 6 per cent. per annum? Ans. 1 dol. 47cts. 2 m.

42. How much is the amount of 455 dollars 52 cents for 47 days, at 6 per cent. per annum?

Ans. 459 dollars 3 cents 9 mills.

NOTE. Work the above questions by the Double Rule of Three Direct.

When the amount, rate and time, are given to find the principal.

RULE. As the amount of 100 dollars, at the rate and time given, is to 100 dollars, so is the amount given to the principal required.

43. What principal put to interest for eight years, at 6 per cent. per annum, will pay a debt of 1000 dollars?

8 years.
6 per cent.
48
100

					$	cts.	mills.		
As 148	:	100	: :	1000	:	675	67	$5\frac{2}{3}\frac{5}{7}$	Ans.

44. What sum being put to interest for twelve years, at 7 per cent. per annum, will amount to 626 dolls. 6 cts.?
Ans. 340 dolls. 25 cts.

45. What sum put to interest for five years, at 6 per cent. per annum, will amount to 728 dollars? Ans. $560.

46. Received 700 dollars as payment in full for a note with four years interest due thereon, at 6 per cent. per annum. How much does the note call for?
Ans. 564 dollars 51 cents $6\frac{4}{3}\frac{}{1}$ mills.

When the amount, time and principal, are given to find the rate.

Rule. As the principal is to the interest for the whole time, so is 100 dollars to its interest for the same time. Divide the interest so found by the time, and the quotient will give the rate per cent.

47. At what rate per cent. will 340 dollars 25 cents amount to 626 dollars 6 cents in twelve years?

$ cts.
626 6
340 25

As 340 25 : 285 81 : : 100 : 84 ÷ 12 = 7 per cent. Ans.

48. At what rate per cent. will 675 dolls. 67 cts. $5\frac{3}{3}\frac{4}{1}$ m. amount to 1000 dollars in eight years? Ans. 6 per cent.

49. Paid 728 dollars in full for a note of 560 dollars, with five years interest due thereon. What was the rate per cent. per annum? Ans. 6 per cent.

50. At what rate per cent. will 500 dollars amount to 725 dollars in nine years? Ans. 5 per cent.

51. At what rate per cent. will 1260 dollars amount to 1638 dollars in four years? Ans. 7½ per cent.

When the principal, amount and rate, are given to find the time.

Rule. Find the interest of the given sum for one year, and then say, as the interest for one year is to one year, so is the interest of the whole sum to the time required.

52. In what time will 100 dollars, or any other sum of money, double itself, at 6, 7, 8, or any other rate per cent. per annum, simple interest?

RULE. As the rate per cent. is to one year, so is 100 dollars to the number of years it will be in doubling itself; or, divide 100 dollars by the rate per cent. and the quotient is the time.

EXAMPLES.

	$		year.		$		
As	6	:	1	: :	100	:	$16\frac{2}{3}$ years.
	7	:	1	: :	100	:	$14\frac{2}{7}$
	8	:	1	: :	100	:	$12\frac{1}{2}$

Ans. $16\frac{2}{3}$ years at 6 per cent. $14\frac{2}{7}$ years at 7 per cent. $12\frac{1}{2}$ years at 8 per cent. and $100, at 5 per cent. will be 20 years in doubling itself.

53. In what time will 340 dollars 25 cents amount to 626 dollars 6 cents, at 7 per cent. per annum?

Ans. 12 years.

54. In what time will 650 dollars amount to 910 dollars, at 8 per cent. per annum? Ans. 5 years.

A TABLE shewing the number of days, from any day in any month, to the same day in any other month, throughout the year.

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Jan.	365	334	306	275	245	274	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
March	59	28	365	334	304	273	243	212	181	151	120	90
April	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	91	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	91	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

Use of the Table.—Suppose you want to know the number of days between the 8th of May, and the 8th of November following. Under the column of May, at the top of the table, look for November in the left-hand column, and you will have 184 days. Again, how many days from the 4th of March, to the 4th of September following? Look in the

top column for March, and in the left-hand column for September, and run your finger along, until you come to the column headed March, and you will find 184 days. And if it be required to find how many days there be from 14th June, to the 20th April, you must find how many days there are, first from the 14th of June to the 14th of April, which you will find to be 304 days; then add the number of days from the 14th to the 20th June, which is 6, to 304, and it will make 310 days. If the days in the given months be different, their difference must be added to, or subtracted from the number found in the table. Thus, from the 14th of June to the 20th of April, is 304+6=310 days; and from the 20th April to the 14th June, is 61 days, less 6, equal 55 days. If the time exceeds a year, 365 days must be added for each year.

Calculating interest on bonds, notes, &c.

RULE 1. Find the interest of the principal from the time the interest first commenced, to the time of the first payment, and add the interest thus found to the principal; and subtract from the whole the payment made, and the remainder forms a new principal, on which proceed as you have done, till all the payments are brought in. By this rule, when a payment alone, or in conjunction with any preceding payment, is less than the interest then due, no calculation is to be made, but these lesser payments added to the next. Therefore no part of the interest ever forms, or becomes a part of the principal bearing interest, the payments being first applied to discharge the interest.

There are other methods used, which I will give examples of; but the above one is the best.

EXAMPLE.

55. A passed his note to B for 800 dollars, dated the first of January, 1823, payable in four years, with interest from the date, at 6 per cent. per annum, on which A paid the following sums, viz.

1823, July 1st, paid on said note	$ 100
1823, Nov. 1st, paid	150
1824, Jan. 1st, paid	100
1824, Sept. 1st, paid	80
1825, Sept. 1st, paid	100
1826, April 1st, paid	150
	$ 680 paid.

What was the balance due on A's note, on the first day of January, 1827?

Principal at interest from Jan. 1st to July 1st, 1823	$800	00
Six months interest	24	00
Due first of July	824	00
Paid same date	100	00
Balance due, the new principal	724	00
Interest on said balance, up to 1st Nov. (4 months)	14	48
Due first of November	738	48
Paid same date	150	00
The new principal, bearing interest	588	48
Interest on said balance, up to Jan. 1st, 1824, (2 months)	5	88+
Due first of January, 1824	594	36
Paid same date	100	00
The new principal, bearing interest	494	36
Interest on said new principal, to Sept. 1st, (8 months)	19	77+
Due first of September, 1824	514	13
Paid same date	80	00
The new principal, bearing interest	434	13
Interest on said new principal, to Sept. 1, 1825, (12 months)	26	04+
Due first of September, 1825	460	17
Paid same date	100	00
The new principal, bearing interest	360	17
Interest on said new principal, to April 1, 1826, (7 months)	12	61+
Due first of April, 1826	372	78
Paid same date	150	00
The new principal, bearing interest	222	78
Interest on said new principal, to Jan. 1, 1827, (9 months)	10	02+
Due first of January, 1827	$232	80

NOTE. I did not bring in the fractions.

Another method, used by merchants.

RULE 2. Multiply the principal by the number of days, till the first payment is made, and the remaining principal by the number of days between the first and second payments, &c. till all the payments are made; then add all these products together, and divide the whole sum by 60, and the quotient will be the answer, or interest in cents. This divisor 60 will do when the rate is at 6 per cent. This 60 is found by dividing 6 into 365 days. A divisor for 5 per cent. is found by dividing 365 by 5=73, &c.

The above question performed by this rule.

			dolls.	days.	days.	$	cts.
1823, Jan.	1st,	Principal	800	× 181 =	144800	23	80
July	1st,	Paid	100				
		Balance	700	× 123 =	86100	14	15+
Nov.	1st,	Paid	150				
		Balance	550	× 61 =	33550	5	51+
1824, Jan.	1st,	Paid	100				
		Balance	450	× 243 =	109350	17	97+
Sept.	1st,	Paid	80				
		Balance	370	× 365 =	135050	22	20+
1825, Sept.	1st,	Paid	100				
		Balance	270	× 212 =	57240	9	40+
1826, April	1st,	Paid	150				
		Balance	120	× 275 =	33000	5	42+
		Interest	98 45				
					599090	98	45

Amount due $218 45

60)599090

9984+ cts.

Balance	$120	00
Interest	.99	84+
Amount due	$219	84

NOTE. The column above, containing the interest of the several balances is calculated by Compound Proportion. The divisor 60 is not correct, because 365 divided by 6 gives 60$\frac{5}{6}$ as a quotient. But multiply 365 days by 100 dollars = 36500 ÷ 6 = 6083+ for a divisor, which will give the quotient in dollars. Thus,

6083)599090

98	43+	interest.
120	00	balance.

Amount due $218 43

RULE 3. As 100 dollars, with 365, is to 6 per cent, so is the different balances, with their number of days, to their respective interests.

RULE 4. Multiply the number of days into the balances at interest, add their products together, and divide that sum by 6083, and the quotient will be the answer in dollars, that is, the interest, which, added to the balance of principal due, will be the whole amount due.

Another method.

Rule 5. When the time is months, or months and days between each payment, multiply each sum by the months it is at interest, and take the quotient of 1200, divided by the rate per cent. as a divisor. Thus, for 6 per cent. the divisor is 200, for 5 per cent. 240, for 8 per cent. 150, &c.

THE SAME QUESTION.

Principal	$800	×	6	= 4800
Paid 1st July, 1823	100			
New principal	700	×	4	= 2800
Paid 1st Nov. 1823	150			
New principal	550	×	2	= 1100
Paid 1st Jan. 1824	100			
New principal	450	×	8	= 3600
Paid 1st Sept. 1824	80			
New principal	370	×	12	= 4440
Paid 1st Sept. 1825	100			
New principal	270	×	7	= 1890
Paid 1st April, 1826	150			
Balance up to Jan. 1, 1827	120	×	9	= 1080
			200)	19710

Interest	98	55
Balance	120	00
Due	$218	55

Another method.

Rule 6. Find the interest on the principal, from the time the interest commenced, to the time of settlement, and add it to the principal, and likewise the interest on each payment, from the time the payment was made, to the time of the settlement, and add the several interests to their respective payments, and deduct the whole amount from the amount of the note, bond, &c. and the remainder is the balance due on the note, &c.

THE SAME QUESTION.

Principal 800 dollars, and four years interest 192 dollars.

Whole amount, up to 1st Jan. 1827				$992 00
Paid 1st July, 1823	$100			
Interest thereon to 1st Jan. 1827	21			
		amount	$121	
Paid 1st Nov. 1823	150			
Interest thereon to 1st Jan. 1827	28	50		
			178	50
Paid 1st Jan. 1824	100			
Interest to 1st Jan. 1827	18			
			118	
Paid 1st Sept. 1824	80			
Interest to 1st Jan. 1827	11	20		
			91	20
Paid 1st Sept. 1825	100			
Interest to 1st Jan. 1827	8			
			108	
Paid 1st April, 1826	150			
Interest to 1st Jan. 1827	6	75		
			156	75
			$773 45	773 45
			Due	$218 55

Another method.

RULE 7. When payments are made in shorter periods than a year, that is, in 4, 6, 7, 8 months, &c. from each other, find the interest on the principal sum for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year, add it to the sum paid, and deduct it from the principal and interest, added as above. If any payment be made of a less sum than the interest due at the time of such payment, no interest is to be calculated, but only on the principal sum, for any period.

THE SAME QUESTION.

1st Jan. 1823, principal sum	$800			
One year's interest, to 1st Jan. 1824	48			
			amount	$848 00
Paid 1st July, 1823	100			
Six months interest, to 1st Jan. 1824	3			
			103	
Paid 1st Nov. 1823	150			
Interest to 1st Jan. 1824, (2 months)	1	50		
			151	50
Paid 1st Jan. 1824			100	
				354 50
			New principal	493 50

	New principal brought over		$ 493	50
One year's interest on 493 dolls. 50 cts. to 1st Jan. 1825			29	61
	Amount		523	11
Paid 1st Sept. 1824		80		
Interest thereon to 1st Jan. 1825, (4 months)		1 60		
			81	60
	New principal		441	51
One year's interest on 441 dolls. 51 cts. to 1st Jan. 1826			26	49
	Amount		468	00
Paid 1st. Sept. 1825		100		
Interest thereon to 1st Jan. 1826, (4 months)		2		
			102	
	New principal		366	00
Interest on 366 dollars, to 1st Jan. 1827			21	96
	Amount		387	96
Paid 1st April, 1826		150		
Interest thereon to 1st Jan. 1827, (9 months)		6 75		
			156	75
	Balance due		$ 231	21

Another method.

Rule 3. Multiply the several balances into the days they are at interest; then multiply the sum of these products by the rate on the dollar, and divide the last product by 365, the days in a year, and you will have the whole interest due, &c.

THE SAME QUESTION.

By the second rule, the sum of the several balances, multiplied into the number of days they were at interest, is

	599090
Ratio, or rate of the dollar, is	,06
	365)35945,40
Interest	98.48+
Balance of principal	120,00
Due	$ 218,48

Different methods.

By Rule 1, due on said note,	$ 232	80+
Rule 2,	219	84
Rule 3,	218	46
Rule 4,	218	48

By Rule 5, due on said note,	$218	55
Rule 6,	218	55
Rule 7,	231	21
Rule 8,	218	48

NOTE. I have here given eight examples, by which interest on notes, where partial payments are made, is computed. The first rule is most in use, and is allowed to be the most just; yet there is a great difference between it and all the rest, save the seventh.

I cannot inform the scholar, whether there is or not in this state, a rule established by law for calculating interest on bonds, notes, &c. where partial payments are made; but in some of the New England states there is.

COMPOUND INTEREST

Is when the interest is added to the principal for one, or at the end of a year, and that sum becomes a new principal for another year, and the interest thereof added again, &c. for any number of years. This is called Compound, or interest upon interest.

RULE 1. Find the interest for one year, and add it to the principal, which is the amount for one year. Find the interest of that amount, which add as before, for the amount of the second year, and so on for any number of years. Subtract the original principal from the last amount, and the remainder will be the compound interest for the whole time.

RULE 2, by the table. Multiply the figures standing against the number of years by the given principal, and the product is the amount required, from which, if you subtract the principal, the remainder is the compound interest.

RULE 3, by the table. Multiply the given principal continually by the amount of one dollar for one year, at the rate per cent. given, until the number of multiplications is equal to the given number of years, and the product will be the amount required, from which, if you subtract the given principal, the remainder will be the compound interest.

Yrs.	5 per cent.	6 per cent.	Yrs.	5 per cent.	6 per cent.
1	1,05000	1,06000	11	1,71034	1,89839
2	1,10250	1,12360	12	1,79585	2,01219
3	1,15762	1,19101	13	1,88565	2,13292
4	1,21550	1,26247	14	1,97993	2,26090
5	1,27628	1,33822	15	2,07893	2,39655
6	1,34009	1,41851	16	2,18287	2,54727
7	1,40710	1,50363	17	2,29201	2,69277
8	1,47745	1,59384	18	2,40661	2,85433
9	1,55132	1,68947	19	2,52695	3,02559
10	1,62889	1,79084	20	2,65329	3,20713

By this table, compound interest may be more expeditiously calculated, in which the amount of one dollar for any number of years, not exceeding 20, is shewn, at 5 and 6 per cent. per annum, compound interest. Look in the left-hand column for the given number of years, and run your eye along towards the right hand, under 5 or 6 per cent. (as the question demands,) and you will find the number sought.

1. How much is the compound interest of 200 dollars for 4 years, at 6 per cent. per annum?

$ 200	1st principal
6	
12,00	interest
200	
212	2nd principal for 1 year
6	
12,72	interest
212	
224,72	3d principal for 2 years
6	
13,48,32	interest
224,72	
238,20,32	4th principal for 3 years
6	
14,29,21,92	interest
238,20,32	
252,49,53,92	amount for 4 years
200	principal
$ 52,49,53,92	answer

Operation by Rule 2.

Tabular number for 4 years, at 6 per cent. 1,26247
200

Amount 252,49,4,00
Principal 200

Answer $ 52,49,4

Operation by Rule 3.

Tabular number for 1 year, 1,06000=1,06
Then 200×1,06×1,06×1,06×1,06=$252,49,5+
Deduct the principal 200

Ans. $ 52,49,5

2. How much is the amount of 500 dollars for 3 years, at 6 per cent. per annum, compound interest?

Ans. 595 dollars 50+ cents.

3. Required the compound interest of 100 dollars for 3 years, at 6 per cent. per annum?

Ans. 19 dollars 10+ cents.

4. What is the amount of 750 dollars for 4 years, at 6 per cent. per annum, compound interest?

Ans. 946 dollars 85+ cents.

5. How much will 650 dollars amount to in 20 years, at 6 per cent. per annum, compound interest?

Ans. 2084 dollars 63+ cents.

6. What is the compound interest of 100 dollars for 11 years, at 6 per cent. per annum?

Ans. 89 dollars 82 cents 9 mills.

7. What is the amount of 425 dollars 75 cents for 3 yrs. at 6 per cent. per annum, compound interest?

Ans. 507 dollars 7+ cents.

8. How much is the amount of 300 dollars for 4 years, at 5 per cent. per annum, compound interest.

Ans. 364 dollars 65 cents.

9. A father left 6000 dollars at compound interest, at 6 per cent. per annum, to be equally divided among three sons, when the youngest, who was 12 years of age, should arrive at 21. What will be each son's share then?

Ans. 3378 dollars 94 cents each.

NOTE. The ratio is the simple interest of one dollar for one year, at any proposed rate per cent. and is thus found.

As 100 : 5 :: 1 to 0,05 ratio.

As 100 : 6 :: 1 to 0,06 ratio.

The table is constructed in the following manner. To find the tabular numbers, say,

As 100 : 105 :: 1 to 1,05 = 1,05000 tabular number.

As 100 : 106 :: 1 to 1,06 = 1,06000 tabular number.

If the pupil does not wish to work by the tabular numbers, he must make a stating for every year in the question, as in simple interest.

COMMISSION, INSURANCE & BROKERAGE.

Commission is an allowance of so much per cent. to a correspondent, for buying and selling goods for his employer, and is performed the same as interest for a year.

RULE. Multiply the given sum by the rate per cent. and divide the product by 100; or cut off two figures to the right hand, and all to the left will be dollars.

EXAMPLES.

1. What is the commission on 300 dollars, at 4 per cent.?

300
4

Ans. $12,00

2. Required my commission on laying out 750 dollars, at 4½ per cent.? Ans. 33 dollars 75 cents.

3. What must I charge my employer on laying out 1260 dollars on his account, at 2½ per cent.?

Ans. 31 dollars 50 cents.

4. What is my commission on 2176 dollars 50 cents, at 2½ per cent.? Ans. 54 dollars 41½ cents.

5. How much am I indebted to my correspondent, on laying out for me 950 dollars 75 cents, when I allow him 2¾ per cent. commission? Ans. 26 dolls. 14½ cts.

INSURANCE

Is a premium given for insuring property of all kinds from fire, the dangers of the sea, even insuring lives, &c. The instrument of agreement between the parties, is called the policy of insurance.

RULE. As in commission, being all at so much per cent.

EXAMPLES.

1. What is the premium of insuring 800 dollars, at 1¼ per cent.? Ans. 10 dollars.

2. What is the insurance on 1240 dolls. at 10 per cent.?

Ans. 124 dollars.

3. What will the insurance of 2800 dollars come to, at 12½ per cent.? Ans. 350 dollars.

4. How much is the insurance of a house, in which there is spiritous liquors, gunpowder and vitriol, (such goods doubly hazardous,) amounting to 89840 dollars, at 31 per cent.? Ans. 27850 dollars 40 cents.

5. What sum must a policy be taken out for, to cover 1000 dollars, when the premium is 10 per cent.?

100 per cent.
10 premium

As 90 : 100 :: 1000 : 1111 dolls. 11 cts. 1⅑ m.

6. What sum will it require to cover a policy of insurance, for 4500 dollars, at 20 per cent.?

100—20=80 As 80 : 100 :: 4500 : $5625 answer.

BROKERAGE

Is an allowance of so much per cent. to persons, for assisting merchants to sell their goods, or procuring customers; and they are generally termed brokers, factors, and by some commission merchants. Their per centage is scarcely 1 per cent. but often $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, &c. This is performed the same as simple interest, commission, &c.

RULE. Multiply the given sum by the rate per cent. and cut off two figures of that product to the right hand, and the left-hand figures will be dollars, and the right-hand figures will be cents. Or, by the Rule of Three, say, as 100 dolls. is to the rate per cent. so is the given sum to the answer, in the same as the rate per cent. is.

EXAMPLES.

1. What is the brokerage on 400 dollars, at $\frac{1}{2}$ per cent.?

$\frac{1}{2})400$

Ans. $2,00

dolls. cts. dolls.
As 100 : 50 : : 400 : 2 dollars.

2. How much must I pay my factor for obtaining customers for me, who purchased 1200 dollars worth tobacco of me, the factorage being $\frac{7}{8}$ per cent.?

Ans. 7 dollars 50 cents.

3. What is the brokerage on 1000 dolls. at $1\frac{1}{2}$ per cent.?

Ans. 15 dollars.

4. How much must I pay factorage at $1\frac{1}{4}$ per cent. on 2000 dollars? Ans. 25 dollars.

REBATE OR DISCOUNT

Is an allowance made of any sum of money before it becomes due, or upon advancing ready money on bonds and notes, which are payable at a future date. What remains after the rebate is deducted, is the present worth, or such a sum, as if put to interest, would, at the given rate and time, amount to the given sum or debt.

RULE. As the amount of 100 dollars, at the given rate and time, is to the interest of 100 dolls. at the same rate and time, so is the given sum to the rebate required. Subtract this rebate from the given sum, and the remainder is the present payment or worth. Or, as the amount of 100 dolls. is to 100 dollars, so is the given sum to the present worth, which subtract from the debt, and the remainder is the rebate.

PROOF. Find the amount of the present worth, at the given time and rate, and if the work is right, that will be equal to the given sum or debt.

EXAMPLES.

1. What is the rebate on 200 dollars, which is not due for a year, at 6 per cent. per annum?

	dolls.	*dolls.*	*dolls.*	*dolls. cts. m.*
100+6=106	As 106	: 100 ::	200	: 188 67 9+

dolls. cts. m.		*dolls. cts. m.*	
$200—188 67 9+	equal	11 32 0+	answer.

2. What is the interest on 188 dollars 67 cents $9\frac{8}{9}$ mills for one year, at 6 per cent. per annum?

Ans. 11 dollars 32 cents $0\frac{3}{9}\frac{2}{5}$ mill.

NOTE. Rebate and interest are often supposed to be the same thing, and the interest for the time is often taken in business for the rebate, and both parties allow that neither of them sustain a loss. This is not correct, as you may see by the two questions above; for the interest on 200 dollars for a year, at 6 per cent. is 12 dollars, whereas the rebate on the same sum, and at the same rate and time, is 11 dollars 32+ cents, making a difference of 67 cents 9+ mills, on every 200 dollars, for a year. On a large sum, for six or seven years, the difference would be very considerable.

3. What ready money must I pay, to discharge a debt of 600 dollars, due four years hence, at 5 per cent. per annum?

Ans. 500 dollars.

4. What is the present worth of 450 dollars, due in six months, at 6 per cent. per annum?

Ans. 436 dollars 89 cents.

5. How much ready money must I pay for a note of 615 dollars 75 cents, due seven months hence, rebate at 4½ per cent. per annum? Ans. 600 dollars.

6. What ready money must I pay, to discharge a debt of 650 dollars, due two years hence, rebate at 7 per cent. per annum? Ans. 570 dollars 16+ cents.

When sundry sums are to be paid at different times.

RULE. Find the rebate, or present worth of each particular payment separately, and add them into one sum, and that will be the answer required.

7. What is the rebate of 1000 dollars, one-half due in six months, and the other half due in six months after, at 6 per cent. per annum? Ans. 42 dollars 86+ cents.

8. I have a bond of 2000 dollars, of which 500 dollars is payable in six months, 800 dollars in one year, and the

balance at the end of three years. How much ready money ought I to get for it at present, allowing rebate at 6 per cent. per annum? Ans. 1833 dolls. 37½ cts.

A short method for finding the rebate of any given sum for months and days.

RULE. Lessen the interest of the given sum for the time, by subtracting its own interest, and this gives the rebate very nearly.

9. What is the rebate of 50 dollars for six months, at 6 per cent. per annum? Ans. 1 dol. 46 cts.

Interest on $50 for six months $1 50 cts.
Interest on 1 dol. 50 cts. six months 4

Ans. $1 46

10. What ready money must I pay for my bond, which has 73 days to go before it is due, for 900 dollars, rebate at 6 per cent. per annum? Ans. 889 dolls. 32+ cts.

NOTE. When rebate is to be made for present payment, without any regard to time, the interest on the sum is calculated for a year, as the discount or rebate.

11. How much is the rebate of 750 dolls. at 4 per cent.? Ans. 30 dollars.

12. How much ready money must I give for 1000 dolls. at 5 per cent.? Ans. 950 dollars.

ANNUITIES BY SIMPLE INTEREST.

An annuity is a sum of money, payable every year, or for a certain number of years, or for ever. When an annuity is bought off, or paid all at once at the beginning of the first year, the price paid is called the present worth.

To find the amount of an annuity.

RULE. Find the interest of the given annuity for one year, and for as many years up to the given time, less one; then multiply the annuity by the given number of years, and add that product to the whole interest, and the sum will be the amount sought.

EXAMPLES.

1. If an annuity of 280 dollars be forborne five years, what will be due for principal and interest at the end of said time, simple interest being computed at 5 per cent. per annum? Ans. 1540 dollars.

Interest on 280 dollars for 1 year, at 5 per cent.		$14
280	2	28
280	3	42
280	4	56
Five years' annuity, at 280 dollars per year		1400
	Ans.	$1540

2. A farm being let upon a lease of four years, at 100 dollars per annum, and the rent being in arrears for the whole term, I demand the sum due at the end of the term, simple interest being allowed at 6 per cent. per annum?
Ans. 436 dollars.

To find the present worth of an annuity at simple interest.

RULE. Find the present worth of each year by itself, rebating from the time it falls due, and the sum of all these will be the present worth required.

3. How much is the present worth of 400 dollars per annum, to continue four years, at 6 per cent. per annum?
Ans. 1396 dollars 6½ cents.

4. If the payment of the rent of a house be omitted for seven years, what will be the amount in that time, at 6 per cent. when the rent is 224 dollars per year?
Ans. 1850 dollars 23+ cents.

5. If the payment of a pension of 200 dollars per year be forborne seven years, what will it amount to, at 4 per cent. per annum? Ans. 1568 dollars.

6. Suppose a salary of 400 dolls. a year be forborne seven years, what is the amount at the expiration of that time, at 4½ per cent. per annum? Ans. 3178 dollars.

7. What ready money will equal an annuity of 100 dollars, to continue three years, rebate being at 6 per cent. per annum? Ans. 268 dollars 37+ cts.

NOTE. Nos. 4, 5 and 6, are wrought by the first rule. When the annuity, salary or rent, are to be paid by half yearly or quarterly payments, the amount of such annuity, salary or rent, will be more than yearly, half yearly more than yearly, and quarterly more than half yearly.

RULE for half yearly payments. Take always half of the ratio, half of the annuity, and twice the number of years; that is, reduce the years to half years, and work as before.

8. If 280 dollars annuity, payable half yearly, be forborne five years, what will it amount to in that time, at 5 per cent. per annum? Ans. 1557 dollars 50 cents.

RULE for quarterly payments. Take a fourth part of the ratio, a fourth part of the annuity, and four times the number of years; that is, reduce the years to quarters, and work as before.

9. If 280 dollars annuity, payable quarterly, was unpaid five years, what will it amount to in that time, at 5 per cent. per annum? Ans. 1566 dollars 25 cents.

To shew the difference of the payments.

280 dollars in	5	years, by yearly payments, is	$ 1540	
280	5	by half yearly payments	1557	50
280	5	by quarterly payments	1566	25

ANNUITIES OR PENSIONS AT COMPOUND INTEREST.

To find the amount of an annuity, or pension, in arrears, by compound interest.

RULE 1. Make 1 the first term of a geometrical progression, and the amount of one dollar for one year, at the given rate per cent. the ratio. Carry on the series up to as many terms as the given number of years, and find its sum, which multiply by the given annuity, and the product will be the amount required.

TABLE 1st.

Yrs.	5	6	Yrs.	5	6
1	1,000000	1,000000	17	25,840366	28,212380
2	2,050000	2,060000	18	28,132385	30,905653
3	3,152500	3,183600	19	30,539004	33,759992
4	4,310125	4,374616	20	33,065954	36,785592
5	5,525631	5,637193	21	35,719252	39,992727
6	6,801913	6,975319	22	38,505214	43,392291
7	8,142009	8,393838	23	41,430475	46,995828
8	9,549109	9,897468	24	44,501999	50,815578
9	11,026564	11,491316	25	47,727099	54,864512
10	12,577892	13,180770	26	51,113454	59,156382
11	14,206787	14,971643	27	54,669126	63,705765
12	15,917126	16,869942	28	58,402583	68,528112
13	17,712982	18,882130	29	62,322712	73,639798
14	19,598632	21,015066	30	66,438847	79,058186
15	21,578564	23,275969	31	70,760790	84,801677
16	23,657492	25,672528			

RULE 2, by Table 1st. Multiply the tabular number, under the rate per cent. and opposite to the given number of years, by the annuity or rent, and the product will be the amount sought. The table shews the amount of one dollar annuity, forborne or unpaid, at 5 and 6 per cent. per annum, compound interest, for 31 years, or under.

TABLE 2nd.

Yrs.	5	6	Yrs.	5	6
1	0,952381	0,943396	17	11,274066	10.477260
2	1,859410	1.833393	18	11,689587	10,827603
3	2,723248	2.673012	19	12,085321	11,158116
4	3,545950	3,465106	20	12,462210	11.469921
5	4 329477	4,212364	21	12.821153	11,764077
6	5,075692	4,917324	22	13,163003	12.041582
7	5,586278	5,582381	23	13,488574	12,303380
8	6,463213	6,209794	24	13,798642	12,550357
9	7,107832	6,801692	25	14,196944	12,783356
10	7,721735	7,360087	26	14,375185	13,003166
11	8,306414	7,886875	27	14,643034	13,210534
12	8,863252	8,383844	28	14,898127	13,406164
13	9,393573	8,852683	29	15,141073	13,590721
14	9,898641	9,294984	30	15,372451	13,764831
15	10,379658	9,712249	31	15,592810	13,929086
16	10,837769	10,105895			

Table 2nd shews the present worth of one dollar annuity, to continue for 31 years, at 5 and 6 per cent. per annum, compound interest.

1. If 150 dollars yearly rent, or an annuity, be forborne or unpaid four years, what will it amount to in that time, at 6 per cent. per annum, compound interest?

Operation by Rule 1.

$1,06 \times 1,06 \times 1,06 =$	1,191016
$1,06 \times 1,06 =$	1,123600
Amount of one dollar for a year	1,060000
First term	1,000000
Sum of the series	4,374616

$	cts.	m.
$4,374616 \times 150 = 656,192400 =$ 656	19	2+ Ans.

NOTE. Look for the amount of one dollar, &c. in the table of compound interest, and you will find 1,06, &c.

By Table 1st, opposite 4 years, and under 6 per cent. per annum, is

$4,374616 \times 150 = 656$ dolls. 19 cts. 2+ m. Ans.

2. If a salary or pension be forborne 20 years, which is 60 dollars, how much will it come to, at 6 per cent. per annum, compound interest? Ans. 2207 dolls. 13+ cts.

3. Suppose an annuity of 400 dollars be 12 years in arrears, it is required to find what is due now, compound interest being allowed at 5 per cent. per annum?

Ans. 6366 dolls. 85+ cts. by table 1st.

4. What will a pension of 480 dollars per annum, payable yearly, amount to in three years, at 5 per cent. per annum, compound interest? Ans. 1513 dolls. 20 cts.

To find the present worth of annuities at compound interest.

RULE. Divide the annuity, &c. by that power of the ratio, signified by the number of years, and subtract the quotient from the annuity. This remainder being divided by the ratio less one, the quotient will be the present value of the annuity required.

5. What ready money will purchase an annuity of 200 dollars, to continue four years, at 5 per cent. compound interest?

Fourth power of the ratio, 1,215506; then divide $200 by 1,215506, subtract that quotient from $200, and divide the remainder by ,05, and you will have 709 dollars 19+ cents, which is the answer.

By Table 2nd.

Under 5, and even with 4 years, you have the number, 3,545950, equal to the present worth of one dollar for four years, which multiplied by the annuity of 200 dollars, will give 709 dollars 19+ cents as the answer.

6. What is the present worth of an annuity of 60 dollars per annum, to continue 20 years, at 6 per cent. compound interest? Ans. 688 dollars 19+ cents.

7. What is 120 dollars per annum, to continue 7 years, worth in ready money, at 6 per cent. compound interest?

Ans. 669 dollars 88+ cents.

To find the present worth of annuities, leases, &c. taken in reversion, at compound interest.

RULE. Divide the annuity by that power of the ratio, denoted by the time of its continuance, subtract the quotient from the annuity, divide the remainder by the ratio less one, and the quotient will be the present worth, to commence immediately. Divide this quotient by that power of the ratio, denoted by the time of reversion, or the time to come, before the annuity commences, and the quotient will be the present worth of the annuity in reversion.

8. What ready money will purchase an annuity of 200 dollars, payable yearly, for four years, but not to commence till two years, at 5 per cent.?

4th power of 1,05=1,215506)200,00000(164,53052
164,53052

1,05—1 = ,05)35,46948

2nd power of 1,05=1,1025)709,188

Present worth of the annuity in reversion $643,25+ answer.

By Table 2nd. Find the present worth of one dollar, at the given rate, for the sum of the time of continuance, and time in reversion. Add these together, and from which value subtract the present worth of one dollar, for the time in reversion. Multiply the remainder by the annuity, and the product will be the answer.

Time of continuance 4 years
Time of reversion 2
6 years give 5,075692
Time in reversion 2 1,859410
Rem. 3,216282
200
$643,25,6,400 answer.

9. What is the present worth of 300 dollars yearly rent, which is not to commence until ten years hence, and then to continue seven years after that time, at 6 per cent.?
Ans. 935 dollars 15 cents.

10. What is the present worth of a reversion of a lease of 60 dollars per annum, to continue 20 years, but not to commence until the end of 8 years, allowing 6 per cent. to the purchaser? Ans. 431 dolls. 78 cts. 2+ m.

To find the present worth of a freehold estate, or an annuity to continue for ever.

RULE. As the rate per cent. is to 100 dollars, so is the annuity or yearly rent to the value required.

11. What is the worth of a freehold estate of 200 dollars per annum, allowing 5 per cent. to the purchaser?

As 5 : 100 :: 200 : $4000 Ans.

12. An estate brings in yearly 600 dollars; what would it sell for, allowing the purchaser 6 per cent. for his money?
Ans. 10000 dollars.

To find the present worth of a freehold estate in reversion.

RULE. Find the present value of the estate by the foregoing rule, as though it was to be entered upon immediately, and divide the said value by that power of the ratio, denoted by the time of reversion, and the quotient will be the present worth of the estate in reversion.

13. A freehold estate, which brings in 160 dollars yearly, is to be sold, but does not commence for two years. What is its value, allowing the purchaser 5 per cent.?

As 5 : 100 : : 160 to $3200, the present worth, if entered on immediately. Then 2nd power of the ratio, denoted by the time of reversion, 1,1025 ÷ 3200 = 2902 dollars 49+ cents, the present worth of 3200 dolls. in two years reversion.

By Table 2nd. Find the present worth of the annuity or rent, for the time of reversion, which subtract from the value of the immediate possession, and you will have the value of the estate in reversion.

Present worth of one dollar for two years				1,859410
Multiplied by the annuity				160
				$ 297,50,5,600
From	$3200	00	0	present worth
Take	297	50	5	present worth of the rent
Ans.	$2902	49	5	

NOTE. 3200 dollars is the value, if immediately entered.

14. If an estate, which brings 90 dollars per year, to commence ten years hence, were to be sold, what would it bring, allowing the purchaser 6 per cent.?

Ans. 837 dollars 39+ cents.

15. Which is most advantageous, an estate of 400 dollars per annum, in a term of 15 years, or the reversion of such an estate for ever, after the said term of 15 years, computing at the rate of 5 per cent. per annum, compound interest?

Ans. The first term of 15 years is better than the reversion for ever afterwards, by 303 dolls. 72+ cts.

PERMUTATION

Is the changing and shewing in how many different positions any given number of things may be placed. To find the number of permutations or changes that can be made of any given number of things, all different from each other.

RULE. Multiply all the terms, from one up to the given number, continually together, and the last product will be the number of changes.

EXAMPLES.

1. Suppose three men in a room, Tom, Jack and Bill; in how many different positions can you place them?

1st	position,	Tom, Jack and Bill.
2nd	position,	Tom, Bill and Jack.
3d	position,	Jack, Tom and Bill.
4th	position,	Jack, Bill and Tom.
5th	position,	Bill, Jack and Tom.
6th	position,	Bill, Tom and Jack.

$1 \times 2 \times 3 = 6$ Ans.

2. How many changes may be rung on 12 bells, and also how long they would be in ringing but once over, supposing 24 changes might be rung in a minute, and the year to consist of 365 days 6 hours?

Ans. { 479001600 changes.
{ 37 years 49 weeks 2 days 18 hours.

3. Seven gentlemen, who were travelling, met together by chance at a certain inn upon the road, where they were so well pleased with their host, and each other's company, that in a frolic they offered him 150 dollars, to stay at that place so long as they, together with himself, could sit every day at dinner in a different order or position. The host, thinking that they could not sit in many different positions, because there were but a few of them, and that himself could make no considerable alteration, he being but one, imagined that he should make a good bargain, and readily, for the sake of a good dinner, and better company, entered into an agreement with them; and so made himself the eighth person. I demand how long they staid at that inn, and how many different positions they sat in?

Ans. They staid till they buried their host and his son, and the time was 110 years 142 days, allowing 365 days 6 hours to the year. The number of positions were 40320.

4. What number of changes will the nine digits allow of?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880$ Ans.

5. In how many different positions can you place the letters that spell *Washington?* Ans. 3628800.

COMBINATION.

Combination of numbers or things, is the shewing how often a less number of things may or can be taken out of a greater, and combined or joined together differently.

RULE. Take 1, 2, 3, 4, &c. up to the number given to be combined; take another rotation downwards, or decreasing by unity, from the number out of which the combinations are to be made. Multiply the first continually for a divisor, and the latter for a dividend, and the quotient will be the answer.

EXAMPLES.

1. How many combinations can be made of 6 letters out of 10? Ans. 924.

$1\times2\times3\times4\times5\times6=720$ divisor
$12\times11\times10\times9\times8\times7=665280$ dividend

720)665280

Ans. 924

2. How many combinations may be made of 7 letters out of 12? Ans. 792.

PROGRESSION.

Consists of two parts, Arithmetical and Geometrical.

ARITHMETICAL PROGRESSION.

Is any rank or quantity of numbers, more than two, increasing by a common excess, or decreasing by a common difference; as 2, 4, 6, 8, &c. is called the ascending series in arithmetical progression, and 8, 6, 4, 2, &c. is called the descending series in arithmetical progression.

A series in progression consists of five parts, viz. *the first term, the last term, number of terms, common difference, and sum of the series.* By having three of these parts given, the other two may be found, which admit of a variety of questions. The numbers which form the series, are termed or called the *progressional terms;* the first and last terms are called the *extremes.*

In a series of even numbers, the sum of the two extremes will be equal to the sum of any two terms, equally distant

from them; as 2, 4, 6, 8, 10, the two extremes being 2+10=12, so 4+8=12; but if the number of terms be odd, the double of the middle term will be equal to any two terms, equally distant therefrom, as 1, 3, 5, 7, 9, 11, 13. The double of the middle term, 7, equal 14, and 3+11=14, and 5+9=14.

Progressions are best understood by Algebra.

The first term, the last term, and the number of terms, given to find the sum of all the terms.

RULE. Multiply the sum of the two extremes by the number of terms, and half that product will be the answer; or multiply the sum of the two extremes by half the number of terms; or multiply half the sum of the two extremes by the whole number of terms, and the product will be the answer.

The five things in progression are numbered as follows:

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

EXAMPLES.

1. How many strokes does a clock strike in 24 hours?

First for 12 hours. 12+1=13×6=78 the number for 12 hours, which multiplied by 2 equal 156 for 24 hours.

2. A man bought 17 yards of cloth, and gave for the first yard 40 cents, and for the last yard two dollars. What did he pay for the cloth? Ans. 20 dollars 40 cents.

3. If 100 apples were placed in a right line, exactly two yards asunder from one another, and the first two yards from a basket, what length of ground will that person travel, who gathers these apples singly, returning with every apple to the basket to put it in? Ans. 11 miles 840 yards.

4. The first term of an arithmetical series is 4, the last 24, and number of terms 12. Required the sum of the series?

24+4=28 sum of the extremes; then 28×12÷2=168 Ans.

5. A merchant bought 19 yards of muslin, and was to pay 20 cents for the first yard, and 7 dolls. 40 cts. for the last yard. What did the whole come to?

Ans. 72 dollars 20 cents.

The first term, the last term, and the number of terms, given to find the common difference.

RULE. Divide the difference of the extremes by the number of terms less one, and the quotient will be the common difference, or fourth term. Or, from the second term subtract the first, and the remainder divided by the third less one gives the fourth.

6. The extremes, (or first and second terms,) are 5 and 33, and the number of terms 15; what is the common difference, or fourth term?

$$33 - 5 \div 15 - 1 = 2 \text{ Ans.}$$

7. A man had nine sons, whose several ages differ alike; the youngest was three years old, and the oldest thirty-five. What was the common difference of their ages?

Ans. 4 years.

8. A man is to travel from Washington to a certain place in 12 days, and to go but 3 miles the first day, increasing every day by an equal number of miles, so that his last day's journey may be 58 miles. What is his daily increase, and the length of the whole journey?

Ans. Daily increase 5 miles, and whole journey 366.

9. A debt is to be discharged at 16 different payments, in arithmetical progression; the first payment is to be 56, and the last 400 dollars. What is the common difference, and sum of the whole debt?

Ans. { 22 dolls. 93¾ cts. common difference.
{ 8648 dollars, the whole debt.

The first term, last term, and common difference, given to find the number of terms; or, which is the same, the first, second and fourth terms given to find the third.

RULE. Divide the difference of the extremes by the common difference, add one to the quotient, and that sum is the number of terms; or, from the second subtract the first, the remainder divide by the fourth, and to the quotient add one, which gives the third.

10. If the extremes be 5 and 47, and the common difference 2, what is the number of terms?

$$47 - 5 \div 2 + 1 = 22 \text{ Ans.}$$

11. A man, going a journey, travelled 5 miles the first day, 45 miles the last day, and each day increased his journey by 4 miles. How many days did he travel, and how far?

Ans. 11 days, and 275 miles distance.

12. A man being asked how many sons he had, said that the youngest was 6 years old, and the oldest 38, and that one was born every four years in his family. How many had he? $38-6=32\div4+1=9$ Ans.

The last term, the number of terms, and the common difference, given to find the first term; or, the second, third and fourth to find the first term.

RULE. Multiply the fourth by the third, made less by one, and the product subtracted from the second gives the first.

13. A man in 12 days went from Washington to a certain town in the country, every day's journey increasing the former by three miles, and the last day he went 48 miles. What was his first day's journey?

$3\times\overline{12-1}=33$; then $48-33=15$ miles, answer.

14. A man takes out of his purse, at eight several times, so many different numbers of dollars, every one exceeding the former by five, and the last was fifty. What was the first? Ans. 15 dollars.

The last term, the number of terms, and the sum of all the terms, given to find the first term; or, the second, third and fifth given to find the first term.

RULE. Divide the fifth by the third, and from the quotient subtract half the product of the fourth, multiplied by the third less one, which gives the first.

15. A man is to receive 1440 dollars, at 12 several payments, each payment to exceed the other by an equal excess of 16 dollars; what is the first payment?

$1440\div12=120$; then $16\times12-1\div2=88$; then $120-88=\$32$ Ans.

The first term, the number of terms, and the common difference, given to find the last term; or, the first, third and fourth given to find the second term.

RULE. Subtract the fourth from the product of the third multiplied by the fourth, and that remainder added to the first gives the second.

16. What is the last number of an arithmetical progression, beginning at 5, and continuing, by an increase of 7, to 35 places?

$35\times7=245-7=238+5=243$ Ans.

GEOMETRICAL PROGRESSION

Is when any rank or series of numbers are increased by one common multiplier, or decreased by one common divisor, as 1, 2, 4, 8, 16, 32, &c. increased by the multiplier 2; and 27, 9, 3, 1, decreased by the divisor 3.

When any number of terms is continued in geometrical progression, the product of the two extremes will be equal to any two means, equally distant from the extremes, as 2, 4, 8, 16, 32, 64, where $64 \times 2 = 128$, and $32 \times 4 = 128$, and $8 \times 16 = 128$. When the number of terms are odd, the middle term, multiplied into itself, will be equal to the two extremes, or any two means, equally distant from the mean, as 2, 4, 8, 16, 32; then $8 \times 8 = 64$, and $2 \times 32 = 64$, and $4 \times 16 = 64$.

The five things in arithmetical progression are to be observed here also.

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference, or ratio.
5. The sum of all the terms.

As the last term, in a long series of numbers, is very tedious to come at by continual multiplication, therefore, for the readier finding it out, there is a series of numbers made use of, called *indices*, beginning with an unit, whose common difference is one. Whatever number of indices you make use of, set as many numbers in geometrical proportion, as is given in the question under them.

As 1, 2, 3, 4, 5, 6 indices.
2, 4, 8, 16, 32, 64 numbers in geometrical proportion.

But if the first term in geometrical proportion be different from the ratio, the indices must begin with a cipher.

As 0, 1, 2, 3, 4, 5, 6 indices.
1, 2, 4, 8, 16, 32, 64 numbers in geometrical proportion.

When the indices begin with a cipher, the sum of the indices made choice of, must be always one less than the number of terms given in the question; for 1 in the indices is over the second term, and 2 over the third, &c. Add any two of the indices together, and that sum will agree with the product of their respective terms.

As in the first table of indices	$2 + 5 = 7$
Geometrical proportion	$4 \times 32 = 128$
And in the second table	$2 + 4 = 6$
Geometrical proportion	$4 \times 16 = 64$

In any geometrical proportion, proceeding from unity, the ratio being known, to find any remote term, without producing all the multiplications attending the operation.

RULE. Find what figures of the indices, added together, will give the exponent of the term wanted; then multiply the numbers standing under such exponent into each other, and it will give the term required.

NOTE. When the exponent 1 stands over the second term, the number of exponents must be one less than the number of terms.

EXAMPLES.

1. A man buys 14 pears, and is to pay 2½ mills for the first, 5 for the second, 10 for the third, &c. in a double geometrical proportion to the last, and is to get the whole, if he pays what the last one came to. How much did the pears cost him?

0, 1, 2, 3, 4, 5 indices or exponents.
1, 2, 4, 8, 16, 32 quarter cents, number of terms.
32 × 32 = 1024 the 10th power of the indices.
8 the 3d power.

Quarter cents 4)8192 the 13th power, being one less than 14.

Ans. 2048 price of the last pear, 20 dolls. 48 cts.

In any geometrical progression, not proceeding from unity, the ratio being given, to find any remote term, without producing all the intermediate terms.

RULE. Find the last term as above, only take care that every product be divided by the first term.

2. A sum of money is to be shared among eight men; the first is to have 80 dollars, and the second 240, and so on, in triple proportion geometrically. What will the last have?

0, 1, 2, 3 indices.
80, 240, 720, 2160 number of terms.

$\frac{2160 \times 2160}{80} = 58320 \times 240 = 13996800 \div 80 = \$174960$ Ans.

3 + 3 + 1 = 7 one less than the number of terms.

3. A man made his will in the following manner: To the children's schoolmaster 50 dollars; his youngest son was to have as much more as the schoolmaster, and each son to exceed the former by as much more. There were nine sons. What was the oldest son's legacy?

Ans. 25600 dollars.

The first term, the last term, or the extremes, and the ratio, given to find the sum of the series.

Rule 1. Multiply the last term by the ratio, and from the product subtract the first term; then divide the remainder by the ratio less one, and the quotient will be the sum of all the terms.

4. If the series in a geometrical question be 2, 6, 18, 54, 162, 486, 1458, and the ratio 3, what is the total amount?

$$\frac{3\times1458-2}{3-1}=2186 \quad \text{answer.}$$

Rule 2, for all such questions. Find the last term, if it is not given, by question 2nd; then subtract the first from it, and divide the remainder by the ratio less one, to the quotient of which add the greater, and that will be the sum required.

Operation of No. 4, by Rule 2.

1458—2=1456÷3—1=728+1458=2186 answer.

5. A young man agrees with a farmer to work 12 days, provided he would give him one-fourth of a cent for the first day, one cent for the second, and four for the third, &c. in quadruple proportion geometrical. What did his wages amount to in 12 days? Ans. 13981 dollars 1+ cent.

6. A man married his daughter on new-year's-day, and gave her husband 20 cents towards her portion, promising, if they would do well, to double it on the first day of every month, for one year. What was her portion?

Ans. 819 dollars.

Note. In working the above question, when putting down the terms under the exponents or indices, say, 1, 2, 4, &c. and the number or sum of your series will be fifths of dollars, which divide by 5, and you will have the answer, as 20 cents equal one-fifth of a dollar.

7. A man bought a horse, and by agreement was to give one grain of corn for the first nail, two for the second, four for the third, &c. There were four shoes, and eight nails in each shoe. I demand how many bushels of corn he paid for said horse, and how much the corn came to, at 25 cents per bushel, allowing 500 grains to make a pint, and 64 pints to a bushel?

Ans. { 134217+ bushels of corn.
Amount 33554 dolls. 25 cts.

8. A crafty young man agreed with a farmer, (ignorant in numbers,) to serve him 12 years, and to have nothing for his labour, but the produce of a grain of wheat for the first year, and that product to be sowed for the second year, and so on from year to year, until the end of said 12 years. I demand what the young man got for his wheat, supposing the increase to be only in a tenfold proportion, at 80 cents per bushel, 7680 grains of wheat to a pint, and 64 pints to a bushel? Ans. 180844 dolls. 80 cts. rejecting remainders.

9. A merchant sold 30 yards of very rich silk velvet, at 2 pins for the first yard, 6 pins for the second, 18 for the third, &c. in triple proportion geometrical. I demand how much the velvet produced, when the pins were afterwards sold, at 400 for a cent? Ans. 5147278302 dolls. 35 cts.

10. The extremes of a geometrical series are 1, and 65536, and the ratio 4; what is the sum of the series?

$65536 \times 4 = 262144 - 1 = 262143 \div 4 - 1 = 87381$ Ans.

NOTE. Always observe, when the first term, (in geometrical proportion,) of the series and the ratio are different, that is, when the first term is either greater or less than the ratio, the indices begin the exponents or indices with a cipher; but if the first term and the ratio be alike, begin your indices with an unit.

11. A man bought 13 geese at market, and was to give apples in pay, and to get all the geese for the quantity of apples the last goose would come to, beginning with the first goose at two apples, and doubling up to the last. How many apples did the last goose come to, as that was what he paid for the whole?

1, 2, 3, 4, 5 indices.
2, 4, 8, 16, 32 terms. } The first term and the ratio are alike.

$5 + 5 + 3 = 13$th term. $32 \times 32 \times 8 = 8192$ apples, answer.

To find the whole amount of apples.

The last term $8192 \times 2 = 16384 - 2 = 16382 \div 2 - 1 = 16382$ answer.

12. A man sold 20 oxen, the first for 3 cents, the second for 9 cents, the third for 27 cents, and so on, in triple proportion geometrical. What did the man get for his oxen?
Ans. 52301766 dollars.

ALLIGATION

Is said, by some authors, to be of two kinds, Medial and Alternate, and by some of three kinds, Medial, Alternate and Partial.

Alligation Medial is when the quantities and prices of several things are given to find the mean price of the mixture compounded of those things.

Rule. As the whole composition is to its total value, so is any part of the composition to its mean price.

To prove your work, find the value of the whole composition at the mean rate, and if it agrees with the total value of the several quantities at their own rates, the work is right.

EXAMPLES.

1. A grocer would mix 20 pounds of sugar, at 8 cents a pound, and 15 pounds at 10 cents a pound, with 30 pounds at 12 cents a pound. What is one pound of this mixture worth? Ans. 10 cents 3+ mills.

20 pounds at	8 cents is	160 cents.
15	10	150
30	12	360
65		670

As 65 : 670 :: 1 : 10 cents 3+ mills, answer.

2. A grocer mixed 19 gallons of rum, at 1 dol. 20 cts. per gallon, and 40 gallons of whiskey at 80 cts. per gallon, and 12 gallons of cider at 60 cents per gallon, together. I demand what a gallon of this composition is worth?

Ans. 87 cents 3+ mills.

3. A farmer mingled 20 bushels of wheat, at a dollar per bushel, and 36 bushels of rye at 60 cents per bushel, with 40 bushels of corn at 40 cents per bushel. I want to know the price of one bushel of this mixture? Ans. 60 cts.

4. Suppose 5 lbs. of gold, of 22 carats fine, 2 lbs. of 21 carats fine, and a pound of alloy, be melted together, what is the quality or fineness of this composition?

Ans. 19 carats fine.

5. A farmer hath several sorts of wheat, viz. one sort at 90 cents, another at 80 cents, and a third at 70 cents per bushel. He would mix an equal quantity of each together. I demand the price of a bushel of this mixture?

Ans. 80 cents.

ALLIGATION ALTERNATE

Is when the rates of several things are given to find such quantities of them, as are necessary to make a mixture, which may bear a rate compounded.

RULE. Write the rates of the simples in a column under each other, reduced into one denomination. Connect or link with a continued line the rate of each simple, which is less than that of the compound, with one or any number of those that are greater than the compound, and each greater rate with one or any number of the less. Place the difference between the mean price, (or mixture rate,) and that of each of the simples, opposite to the rates with which they are connected. If then only one difference stands against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

EXAMPLES.

1. How many pounds of tea, at 140 cts. a pound, 150 cts. a pound, and 190 cents a pound, must be mixed together, that one pound may be sold for 170 cents?

$$170 \left\{ \begin{matrix} 140 \\ 150 \\ 190 \end{matrix} \right\} \begin{matrix} 20 \text{ at } 140 \text{ cts.} \\ 20 \text{ at } 150 \text{ cts.} \\ 30+20=50 \text{ at } 190 \text{ cts.} \end{matrix}$$

Ans.

NOTE. Questions in this rule admit of a variety of answers, owing to the manner of linking or connecting them. Observe again, suppose you reject the ciphers, or units' place of the above answer, and say, 2 pounds at 140, 2 at 150, and 5 at 190 cents per pound, will do; or, if you add another cipher, and say, 200 pounds at 140 cents, 200 at 150, and 500 at 190 cents per pound, all will answer; or any three numbers in the same proportion to each other, as the above 20, 20, and 50, have to themselves.

This rule is the proof of Alligation Medial.

2. How much wheat at 80 cents, rye at 60 cents, and buckwheat at 40 cts. per bushel, will make a mixture worth 50 cents per bushel?

Ans. 10 of wheat, 10 of rye, and 40 of buckwheat.

3. A grocer mixed three sorts of sugar together, viz. one sort at 10 cents per pound, another at 7 cents, and a third at 6 cents. How much of each sort must he take, that the whole mixture may be sold at 8 cents per pound?

Ans. 3 lbs. at 10 cts. 2 at 7, and 2 at 6.

4. A distiller hath several sorts of whiskey, viz. one sort at 80 cents per gallon, another at 70 cts. another at 60 cts. and a fourth at 40 cents, and he is desirous to mix so much of each sort together, that the whole may be at 50 cents a gallon. I demand how much of each sort he must take?

Ans. 10 gallons at 80 cents, 10 at 70 cts. 10 at 60 cts. and 60 at 40 cents per gallon.

5. A goldsmith has gold of 17, 18, 22 and 24 carats fine; how much must he take of each sort, to make it 21 carats fine? Ans. 3 of 17, 1 of 18, 3 of 22, and 4 of 24 carats fine.

ALLIGATION PARTIAL,

By some called Alternation, is when the rate of all the things, the quantity of but one of them, and the mean rate of the whole mixture, are given to find the several quantities of the rest, in proportion to the quantity given.

RULE. Take the difference between each rate and the mean rate, and place them alternately, as before; then say, as the difference of the same name, with the quantity given, is to the rest of the differences severally, so is the quantity given, to the several quantities required.

EXAMPLES.

1. A grocer would mix 40 pounds of sugar, at 21 cents a pound, with some at 19, 13, and 11 cts. a pound: how much of each sort must he take to mix with the 40 pounds, that he may sell the mixture at 17 cents a pound?

17 { 11 — 4 ; 13 — 2 ; 19 — 4 ; 21 — 6 } against the price of the given quantity.

As 6 : 40 :: 4 : $26\frac{2}{3}$ at 11 cents. Ans.
6 : 40 :: 2 : $13\frac{1}{3}$ at 13 cents.
6 : 40 :: 4 : $26\frac{2}{3}$ at 19 cents.

2. A farmer would mix 10 bushels of wheat, at 65 cents per bushel, with rye at 43 cents, corn at 31 cts. and barley at 25 cents per bushel, so that a bushel of this mixture may be sold for 33 cents. What quantity of each must he take?

33 { 65 — 8 stands against the given quantity. ; 43 — 2 ; 31 — 10 ; 25 — 32 }

As 8 : 10 :: 2 : $2\frac{1}{2}$ bushels of rye. Ans.
8 : 10 :: 10 : $12\frac{1}{2}$ bushels corn.
8 : 10 :: 32 : 40 bushels barley.

3. How many gallons of water must be mixed with wine worth 60 cts. per gallon, so as to fill a vessel of 100 gallons, that a gallon may be sold at 50 cents?

50 { 0 — 10 ; 60 — 50 } 60

As 60 : 100 :: 10 : $16\frac{2}{3}$ gallons water.
60 : 100 :: 50 : $83\frac{1}{3}$ gallons wine.

Ans. $16\frac{2}{3}$ galls. water, and $83\frac{1}{3}$ galls. wine.

4. A grocer would mix sugar at 4 cents, 6 cents, 9 cents, and 11 cents per pound, and he would make a mixture of 240 pounds, so that it might be afforded at 8 cents a pound. What quantity of each did he take?

Ans. 96 lbs. at 11 cts. 72 lbs. at 4 cts. 48 lbs. at 9 cts. and 24 lbs. at 6 cts. per pound.

5. A retailer of groceries has four sorts of tea, at 20 cts. 60 cts. 120 cts. and 200 cts. per pound. The worst would not sell, and the best was too high for the common people; he therefore mixed 120 pounds, and so much of each sort, as to sell it at 80 cents per pound. How much of each sort did he take?

				lbs.	*cts.*
80 { 20, 60, 120, 200 }	120	As 240 : 120 :: 120	:	60 at 20	Ans.
	40	240 : 120 :: 40	:	20 at 60	
	20	240 : 120 :: 20	:	10 at 120	
	60	240 : 120 :: 60	:	30 at 200	
	240				

6. A goldsmith had two sorts of silver bullion, one of 11 oz. fine, and the other of 6 oz. fine, and having determined to mix a pound of it, so that it shall be 9 oz. fine, how much of each sort must he take?

9 { 11, 6 }	3	As 5 : 12 :: 3 to $7\frac{1}{5}$ of 11 oz.	Ans.
	2	5 : 12 :: 2 to $4\frac{4}{5}$ of 6 oz.	
	5		

7. A grocer hath sugars at 9 cents, 17 cents, and 25 cents per pound, and would make a composition of 2 cwt. 16 lbs. worth 21 cents a pound. What quantity of each must he take? Ans. 40 lbs. at 9 cts. 40 at 17, and 160 at 25 cts.

8. How much gold of 16, 18, 19, and 23 carats fine, must be mixed together, to form a mixture of 3 lbs. 4 oz. of 21 carats fine?

Ans. 5 oz. of 16, of 18, of 19, and 25 of 23 car. fine.

9. A distiller has whiskey at 70 cents, and at 115 cents per gallon, and has a mind to mix a quantity together, so that a hogshead of 63 gallons may be sold for 50 dollars 40 cents. How many gallons must be taken of each?

Ans. 14 gallons at 115 cents, and 49 at 70 cents.

10. A grocer would mix teas at 240 cents, 200 cents, and 120 cents per pound, with 20 pounds at 80 cents per pound. How much of each sort must he take, to make the mixture worth 160 cents per pound?

Ans. 20 lbs. at 80 cts. 20 at 240 cts. 10 at 200 cts. and 10 at 120 cents per pound.

11. A man who made cider had three sorts, one sort at 10 cts. per gallon, another at 8 cts. and the third at 6 cts. He would have a composition of 30 gallons, worth 7 cents a gallon. I demand how much of each sort he must take?

Ans. 5 gallons at 10 cents, 5 at 8 cts. and 20 at 6 cts.

12. A vintner hath four sorts of wine, one sort at 2 dolls. per gallon, another sort at 1 dol. 60 cts. another at 1 dollar 20 cents, and a fourth worth but 80 cents. I demand how much of each sort he must take, to make a mixture of 60 gallons, worth 180 cents per gallon.

Ans. 5 gallons of 160 cents, 5 of 120 cents, 5 of 80 cts. and 45 of 200 cents per gallon.

SECTION VI.

EXCHANGE.

By this rule, import and export merchants know what sum of money ought to be received in one country, for any sum of different specie paid in another, according to the given course of exchange between the countries so trading.

To reduce the monies of foreign countries to that of the United States, see the following tables and examples.

ENGLAND AND IRELAND

Keep their books and accounts in pounds, shillings, pence and farthings.

4 Farthings	*(qrs.)*	make	1 Penny. *(d.)*
12 Pence			1 Shilling. *(s.)*
20 Shillings			1 Pound. *(l.)*

A pound sterling is equal by law to 4 dollars 44 cents United States' money, and one pound Irish is also equal by law to 4 dollars 10 cents United States' money.

NOTE. This difference of 34 cents per pound sterling, is owing to the difference of exchange between England and Ireland, as 100*l.* English money is equal to 108$\frac{1}{3}$*l.* Irish money, and the difference is called the par of exchange, which is 8*l.* 6*s.* 8*d.*

FRANCE

Keep their books and accounts, as in the following table:

12 Deniers	make	1 Sol.
20 Sols		1 Livre.

A livre is equal to 18$\frac{1}{2}$ cents of the currency of United States' money.

SPAIN

Keep their books and accounts, as in the following table:

34 Marvadies of plate make	- - - - -	1 Rial.
8 Rials	- - - - - - - - - -	1 Piastre.

A piastre is equal to 80 cents United States' money.

HAMBURGH IN GERMANY

Keep their books and accounts, as in the following table:

12 Deniers lubs make	- - - - - - -	1 Sous lubs.
16 Sous lubs	- - - - - - - - -	1 Marks lubs.
3 Marks lubs	- - - - - - - - -	1 Rix dollar.

A Rix dollar is equal to 100 cents, or a dollar United States' money.

THE UNITED NETHERLANDS

Keep their books and accounts, as in the following table:

8 Phennings make	- - - - - - - -	1 Groat.
2 Groats	- - - - - - - - - -	1 Stiver.
20 Stivers	- - - - - - - - - -	1 Guilder.

A guilder is equal to 39 cents United States' money.

PORTUGAL

Keep their books and accounts, as in the following table:

1000 Reas make	- - - - - - - - -	1 Millrea.

A millrea is equal to 1 dollar 24 cents United States' money.

EAST-INDIA MONEY.

CHINA

Keep their books and accounts, as in the following table:

10 Cash make	- - - - - - - - -	1 Candareen.
10 Candareens	- - - - - - - - -	1 Mace.
10 Mace	- - - - - - - - - -	1 Tale.

A tale is equal to 1 dol. 48 cts. United States' money; a pagodas of India is equal to 1 dol. 94 cts. and a rupee of Bengal is equal to 55½ cents.

EXAMPLES.

1. Shipped a cargo of goods from Philadelphia to Liverpool, amounting, as per invoice, to 3860 dollars; how much sterling money will my correspondent credit me for in his books?

cts.	*l.*	*cts.*	*l.*	*s.*	*d.*
As 444 :	1 : :	386000 :	869	7	4+ Ans.

2. Received from my correspondent in Liverpool, a cargo of goods, amounting, as per invoice, to 1000*l.* sterling; how much of this country's money am I indebted to my correspondent for said goods?

As 1*l.* : 444 *cts.* :: 1000*l.* : $4440 Ans.

3. B. H. Everard, of Sligo, (Ireland,) shipped a cargo of Irish linen to John Agnew, of New-York, amounting, as per invoice, to 980*l.* How many dollars will John Agnew give B. H. Everard credit for in his books?

As 1*l.* : 410 *cts.* :: 980*l.* : $4018 Ans.

4. John Agnew, of New-York, shipped a cargo of flaxseed and white-oak staves to B. H. Everard, of Sligo, amounting, as per invoice, to $980 dolls. How much Irish money will B. H. Everard give John Agnew credit for in his books?

As 410 *cts.* : 1*l.* :: $98000 *cts.* : 970*l.* 14*s.* 7*d.*+ Ans.

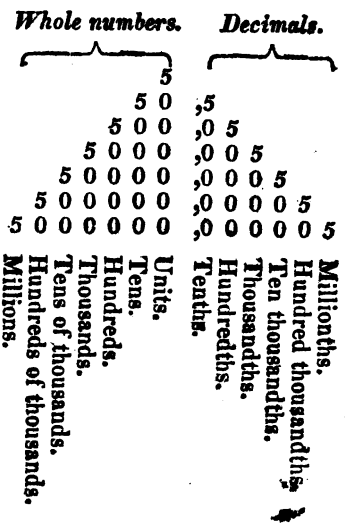
DECIMALS.

A Decimal Fraction has always an unit for its denominator, with a cipher or ciphers annexed to it, as there are figures in the numerator, which being understood need not be expressed, for the true value of the fraction may be expressed by writing the numerator only, with a dot on the left hand; thus, $\frac{5}{10}$ is ,5 and $\frac{25}{100}$ is ,25 &c.

But if the numerator has not so many places as the denominator has ciphers, put as many ciphers before it, at the left hand, as will make up the deficiency; thus, $\frac{5}{100}$ is ,05 and $\frac{5}{1000}$ is ,005 &c. The point prefixed is called the *separatrix*, as it serves to distinguish the fraction, and also separates it from the whole number.

Decimal fractions are always counted from the left hand, towards the right, and each is estimated by its distance from the unit's place, or the first figure of the denominator, towards the left hand; thus, ,2 is two-tenths, ,3 is three-tenths, ,05 is five hundredths, ,005 is five thousandths, &c.

Ciphers, placed to the right hand of decimals, make no alteration in their value, for ,5 ,50 ,500 &c. are all equal to $\frac{1}{2}$, and have the same value; but when prefixed, or put before a decimal, they decrease the fraction in a tenfold proportion, as in the following table.



ADDITION.

Place the numbers exactly under each other, according to the value of their places; then add as in whole numbers, and point off as many places to the right hand, for the decimal fraction, as are equal to the greatest number of decimals in any of the given numbers.

EXAMPLES.

4,5	27,3	8,75
3,05	49,65	19,113
76,070	76,0106	36,8945
14,0006	17,3067	20,90765
97,6206		

4. Add ,7569 + ,25 + ,654 + ,199 parts of a dollar together. Ans. 1,8599 dollars.

5. Add 72,5 + 32,071 + 2,1574 + 371,4 + 2,75 acres together. Ans. 480,8784 acres.

6. Add 71,467 + 27,94 + 16,084 + 98,009 + 86,5 days together. Ans. 300 days.

7. Add twenty-five hundredths, three hundred and sixty-five thousandths, six-tenths, and nine millionths together. Ans. 1,215009.

SUBTRACTION.

RULE. Place the numbers according to their value, and then proceed as in subtraction of whole numbers, and dot off the decimals as in addition.

EXAMPLES.

From	7,05	84,61471	980,0005
Take	3,176	14,00062	374,14654
Rem.	3,874	70,61409	605,85406
From	300	740,2	397,00075
Take	,009	476,06547	1,000007

MULTIPLICATION.

RULE. Multiply as in whole numbers, and from the product point off as many figures for decimals to the right hand, as there are decimals in both factors; but if the product will not admit of so many, supply the defect by prefixing ciphers.

EXAMPLES.

Multiply	36,5	29,831	3,456
By	7,17	,952	,05
Ans.	265,355		
Multiply	76,05	397,006	1,87432
By	3,754	5,6	,005
Multiply	,8765	,0563	,07
By	,0005	,5	,3658

DIVISION.

RULE. Divide as in whole numbers, but observe that the decimal places of the quotient and divisor be equal in number with those in the dividend; and from the right hand of the quotient point off as many places for decimals, as the decimal places in the dividend exceed those in the divisor. If the places in the quotient be not so many as the rule requires, prefix as many ciphers to the left hand of the

quotient as will supply the defect; and if the decimal places in the divisor be more than those in the dividend, annex as many ciphers to the dividend as will make it equal to the divisor. If there happens to be a remainder, you may annex ciphers to it, and carry on the quotient to any degree of exactness you please.

EXAMPLES.

1. Divide ,863972 by 91. Ans. ,009494.

```
91),863972(,009494  Ans.
   819
   ---
   449
   364
   ---
   857
   819
   ---
   382
   364
   ---
    18
```

2	Divide	6	by	4	Ans.	1,5
3		10		3		3,3333+
4		7		2		3,5
5		9		,9		10
6		4,18		,1812		,23068+
7		1,28		8,31		,154+
8		222		,365		608,21+
9		,803		,22		3,65
10		80,3		2,2		36,5
11		222		365		,00821+
12		780,517		24,3		32,12
13		,56		1,12		,5
14		1		12		8,333+
15		72,1564		,1347		535,68+
16		,125		,0457		2,735+
17		1		282		,0035461+

NOTE. When decimals, or whole numbers, are to be divided by 10, 100, 1000, &c. (with ciphers annexed to unity,) it is performed by removing the separatrix in the dividend so many places towards the left hand, as there are ciphers in the divisor.

18	Divide	672	by	10	Ans.	67,2
19		672		100		6,72
20		672		1000		,672

CONTRACTIONS OF DECIMALS.

MULTIPLICATION CONTRACTED.

RULE. Write the unit's place of the multiplier under that of the multiplicand, which place you intend to keep in the product; then invert the order of all the other figures, that is, write them all the contrary way. In multiplying, always begin at that figure in the multiplicand, which stands over the figure you are then multiplying withal, and set down the first figure of each particular product directly one under the other; but yet a due regard must be had to the increase arising from the figures on the right hand of that figure in the multiplicand, at which you begin to multiply. This will appear more plain by the following

EXAMPLE.

1. Multiply 2,368645 by 8,2175, and let there be only four places of decimals retained in the product.

According to the directions, first write down the multiplicand, and under it write the multiplier thus: Place the 8, it being the unit's place of the multiplier, under 4, the fourth place of decimals in the multiplicand, and write the rest of the figures quite contrary to the usual way, as in the following work; then begin to multiply, first the 5 which is left out, (only with regard to the increase, which must be carried from it,) saying, 8 times 5 is 40, carrying 4 in your mind, and 8 times 4 is 32, and 4 I carry is 36; set down 6, and carry 3, and proceed through the rest of the figures, as in common multiplication. Then begin to multiply with 2, saying, 2 times 4 is 8, for which I carry 1, (because it is above 5,) and 2 times 6 is 12, and 1 that I carry is 13; set down 3, and carry 1, and proceed through the rest of the figures. Then multiply with 1, saying, once 6 is 6, for which carry 1, and once 8 is 8, and 1 is 9, which set down, and proceed. Then multiply with 7, saying, 7 times 8 is 56, and carry 6, (because it is above 55,) and 7 times 3 is 21, and 6 that I carry is 27; set down 7, and carry 2, and proceed. Then multiply with 5, saying, 5 times 3 is 15, for which carry 2, and 5 times 2 is 10, and 2 I carry is 12, which set down, and add all the products together, and you will have 19,6107. See the work.

Multiply	2,38645
By	5712,8
	19,0916
	4773
	239
	167
	12
Ans.	19,6107

In multiplying the figure left out every time, next the right hand, is the multiplicand, if the product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3.

I will work the same question the common way, by which you may see all the figures on the right hand of the black line wholly omitted.

Three figures are sufficient to express a decimal fraction.

Multiply	2,98645
By	8,2175
11	93225
167	0515
238	645
4772	90
190916	0
19,6106	52875

2. Let 375,13758 be multiplied by 16,7324, so that the product may have but four places of decimals.

375,13758	multiplicand.
4237,61	multiplier reversed.
37513758	product with 1
22508255	product with 6 increased with 6×8
2625963	product with 7 increased with 7×5
112541	product with 3 increased with 3×7
7503	product with 2 increased with 2×3
1500	product with 4 increased with 0
6276,9590	answer.

Let the same example be repeated, and let only one place in decimals be pricked off.

375,13758	multiplicand.
4237,61	multiplier reversed.
37514	product by 1 with the increase of 1×7
22508	product with 6 increased with 6×3
2626	product with 7 increased with 7×1
119	product with 3 increased with 3×5
7	product with 2 increased with 2×7
1	the increase only of 4×3
6276,9	answer the same as before.

DIVISION CONTRACTED.

RULE. Find what is the value of the first figure in the quotient; then, by knowing the first figure's denomination, you may have as many, or as few places of decimals as you please, by taking as many of the left-hand figures of the divisor as you think convenient for the first divisor, and then take as many figures of the dividend as will answer them, and in dividing omit one figure of the divisor at each following operation.

EXAMPLE.

1. Divide 721,17562 by 2,257432, and let there be three places of decimals in the quotient.

Demonstration.—In this example, the unit's place of the divisor falls under the hundred's place in the dividend, and it is required that three places of decimals be in the quotient; so there must be six places in all, that is, three places of whole numbers, and three of decimals. Then, because I have the divisor in the first six figures of the dividend, I cut off the 62 as useless, and seek how often the divisor is in the dividend, and the answer is 3 times, which put in the quotient, and multiply and subtract as in common division, and the remainder is 43946. Then prick off the 3 in the divisor, and seek how often the remaining figures may be had in 43946, the remainder, which can be but once; put 1 in the quotient, and multiply and subtract, and the next remainder is 21372. Then prick off the 4 in the divisor, and seek how often the remaining figures may be had in 21372, which will be 9 times, which put in the quotient, and multiply thus, saying, 9 times 4 is 36, for which I carry 4, (in respect of the 4 last pricked off,) and 9 times 7 is 63, and 4 is 67; set down 7, and carry 6, and so proceed till the division be finished, always respecting the increase made from the figure pricked off. See the work.

2,257432)721,175|62(319,467 Ans.
677229

43946
22574

21372
20317

1055
903

152
135

17
15

2

THE COMMON WAY.

2,25743)721,17562(319,467 Ans.
677229

43946	6
22574	3
21372	32
20316	87
1055	450
902	972
152	4780
135	4458
17	03220
15	80201
Rem. 1	23019

NOTE. I have set down the work of the last example at large, according to the common way, that thereby the learner may see the reason of the rule, on the right side of the perpendicular black line, being wholly omitted.

REDUCTION OF DECIMALS.

To reduce a vulgar fraction to its equivalent decimal,

RULE. Annex ciphers to the numerator, and divide by the denominator, and the quotient will be the decimal required. As many ciphers as you annex to the given numerator, so many places must be pointed in the quotient; and if there be not so many places of figures in the quotient, make up the deficiency by placing ciphers to the left hand of the said quotient.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to a decimal.

8)1,000

,125 Ans.

2. What decimal is equal to $\frac{1}{4}$?	Ans.	,25
3. What decimal is equal to $\frac{1}{3}$?		,3333+
4. What decimal is equal to $\frac{2}{3}$?		,6666+
5. What decimal is equal to $\frac{1}{2}$?		,5
6. What decimal is equal to $\frac{3}{4}$?		,75
7. What decimal is equal to $\frac{7}{8}$?		,875
8. What decimal is equal to $\frac{2}{5}$?		,4

To reduce quantities of several denominations to a decimal.

RULE 1. Bring the given denomination first to a vulgar fraction, and reduce said vulgar fraction to its equivalent decimal.

RULE 2. Place the several denominations above each other, letting the highest denomination stand at the bottom, and then divide each denomination, (beginning at the top,) by its value in the next denomination, and the last quotient will give the decimal required.

EXAMPLES.

1. Reduce 20 cents to the decimal of a dollar.

$\frac{20}{100} = \frac{1}{5}$ Then 5)1,0

Ans. ,2

2. Reduce 30 cts. to the decimal of a dollar.	Ans.	,3
3. Reduce 36 cts. to the decimal of a dollar.		,36
4. Reduce 2 dollars to the decimal of an eagle.		,2
5. Reduce 2 cents to the decimal of a dime.		,2
6. Reduce 6 dimes to the decimal of an eagle.		,6

7. Reduce 5 mills to the decimal of a dime. Ans. ,5
8. Reduce 38 cents to the decimal of an eagle.
Ans. ,038
9. Reduce 4 dollars 25 cents to the decimal of an eagle.
Ans. ,425
10. Reduce 3 qrs. 2 na. to the decimal of a yard.
Ans. ,875
11. Reduce a gallon to the decimal of a hogshead.
Ans. ,015873+
12. Reduce 2½ feet to the decimal of a yard.
Ans. ,83333+
13. Reduce 3 qrs. 21 lbs. to the decimal of an cwt.
Ans. ,9375
14. Reduce 4 cwt. 2 qrs. to the decimal of a ton.
Ans. ,225
15. Reduce 14 drams to the decimal of a pound Avoirdupois.
Ans. ,0546875
16. Reduce 14 cwt. to the decimal of a ton. Ans. ,7
17. Reduce 7 minutes to the decimal of a day.
Ans. ,0048611+
18. Reduce 2 days to the decimal of a week.
Ans. ,2857142+
19. Reduce a pint to the decimal of a gallon.
Ans. ,125
20. Reduce 76 yards to the decimal of a mile.
Ans. ,04318181+
21. Reduce 4 perches to the decimal of an acre.
Ans. ,025
22. Reduce 4 inches to the decimal of a yard.
Ans. ,1111111111+
23. Reduce 72 days to the decimal of a year.
Ans. ,1972602+

To find the value of a decimal, in the known parts of an integer.

Rule. Multiply the decimal by the number of parts in the next less denomination, and cut off as many places for a remainder, to the right hand, as there are places in the given decimal. Multiply the remainder by the next inferior denomination, and cut off a remainder as before, and so on through all the parts of the integer, and the several denominations standing on the left hand make the answer.

EXAMPLES.

Q. What is the value of ,875 of a yard?
Ans. 3 qrs. 2 na.

,875
4 nails

Qrs. 3,500
4 quarters

Nails 2,000

Ans. 3 qrs. 2 na.

2. What is the value of ,617 of an cwt.?
Ans. 2 qrs. 13 lbs. 1 oz. 10 drs.
3. What is the value of ,875 of a hogshead of wine?
Ans. 55 gallons 1 pint.
4. What is the value of ,712 of a furlong?
Ans. 23 poles 2 yds. 1 ft. 11,04 inc.
5. What is the value of ,17 of a tun of wine?
Ans. 42 gallons 3,36 quarts.
6. What is the value of ,7 lb. of silver?
Ans. 8 oz. 8 dwt.
7. What is the proper quantity of ,761 of a day?
Ans. 18 hrs. 15 min. 53,4 sec.
8. What is the proper quantity of ,71 of 4 oz. of gold?
Ans. 2 oz. 16 dwt. 19,2 grs.
9. What is the proper quantity of ,4712 of an ell English?
Ans. 2 qrs. 1,424 na.
10. What is the value of ,67 of a league?
Ans. 2 miles 3 poles 1 yd. 3+ inc.
11. What is the value of ,12785 of a year?
Ans. 46 days 15 hrs. 57 min. 57+ sec.
12. What is the proper quantity of ,3 of a year?
Ans. 109 days 12 hours.
13. What is the proper quantity of ,5 of an hour?
Ans. 30 minutes.

Practical Questions.

1. What is the sum of ,17 ton ,19 cwt. ,17 qr. ,7 lb.?
Ans. 3 cwt. 2 qrs. 15,54 lbs.
2. What is the sum of ,17 lb. Troy, and ,84 oz.
Ans. 2 oz. 17 dwts. 14,4 grs.
3. What is the difference between ,41 of a day, and ,16 of an hour?
Ans. 9 hrs. 40 min. 48 sec.

EXTRACTION OF THE SQUARE ROOT.

To find the root thereof, that is, to find out such a number, as being multiplied into itself, the product shall be equal

to the number given. Such operation is called Extraction of the Square Root, to perform which observe the following

RULE. 1st. You must point your given numbers, that is, make a point or dot over the unit's place, another over the hundred's, and upon every second figure throughout.

2nd. Then seek the greatest square number in the first point, towards the left hand, placing the square number under the first point, and the root thereof in the quotient, and subtract the said square number from the first point, and to the remainder bring down the next point, and call that the *resolvend.*

3d. Then double the quotient, and place it as a divisor on the left hand of the resolvend, and seek how often the divisor is contained in the resolvend, (always rejecting the unit's place,) and put the answer in the quotient, and also on the right-hand side of the divisor; then multiply by the figure last put in the quotient, and subtract the product from the resolvend, (as in common division,) and bring down the next point to the remainder, (if there be any more,) and proceed as before.

A Table of Squares, Cubes, and their Roots.

ROOTS.	1	2	3	4	5	6	7	8	9
SQUARES.	1	4	9	16	25	36	49	64	81
CUBES.	1	8	27	64	125	216	343	512	729

EXAMPLES.

1. Let 4489 be a number given, the square root of which is required?

```
  ·  ·
  4489(67   Ans.
  36
  ----
127)889   resolvend.
    889   product.
    ---
```

Demonstration.—First point the given number, as before directed, and then, by the above table, seek the greatest square number in 44, (the first point to the left hand,) which you will find to be 36, and 6 the root. Put 36 under 44, and 6 in the quotient, and subtract 36 from 44, and there remains 8, to which bring down the other point, 89, placing it on the right hand, so as to make 889 for a resolvend; then double the quotient 6, and it makes 12, which place on the left hand for a divisor, and seek how often 12 is in 88, (reserving the unit's place,) the answer is 7 times, which put in the quotient, and also on the right hand of the divisor, and multiply 127 by 7, as in common division, and the product is 889, which subtracted from the resolvend there remains nothing, and your work is finished. The square root of 4489 is 67, which root, if multiplied by itself, that is, 67 by 67, the product will be 4489, equal to the given square number. Work all the rest according to this demonstration.

2. Let 106929 be a square number, whose root is required?
Ans. 327.

NOTE. If remainders occur, you may annex ciphers, by two at a time, to the remainder, and so prosecute the work to any degree of exactness, or to as many places of decimals as you please.

3. Required the square root of 144? Ans. 12.
4. Required the square root of 2176782336? Ans. 466,56
5. Required the square root of 1296? 36
6. Required the square root of 56644? 23,8
7. Required the square root of 119025? 345
8. Required the square root of 2268741? 1506,23+
9. Required the square root of 36373961? 6031
10. Required the square root of 43046721? 6561

If the given number be a mixed number, consisting of a whole number and a decimal together, make the number of decimal places even, as 2, 4, 6, 8, 10, &c. that so there may a point fall on the unit's place of the whole numbers.

11. Required the square root of 656714,37512?
Ans. 810,379.

NOTE. In this question there are five places of decimals, therefore you must put a cipher on the right side of the 2, to make it even, so that the point may fall on 4, the unit's place of the whole number.

12. Required the square root of 751417,5745?
Ans. 866,4.
13. Required the square root of 761,801216?
Ans. 27,6007+
14. Required the square root of 3,17218112?
Ans. 1,78106+

To extract the square root of a vulgar fraction.

RULE. Reduce the fraction to its lowest terms, and then extract the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator. If the fraction be a surd, that is, a number whose root cannot be exactly found, reduce it to a decimal, and extract the root from it.

15. What is the square root of $\frac{16}{81}$? Ans. $\frac{4}{9}$
16. What is the square root of $\frac{4}{9}$? $\frac{2}{3}$
17. What is the square root of $\frac{36}{49}$? $\frac{6}{7}$
18. What is the square root of $\frac{25}{64}$? $\frac{5}{8}$

SURDS.

19. What is the square root of $\frac{387}{735}$? Ans. ,72414+
20. What is the square root of $\frac{478}{849}$? ,93308+
21. What is the square root of $\frac{357}{476}$? ,86602+

To extract the square root of a mixed number.

RULE. Reduce the fractional part of the mixed number to its lowest terms, and the mixed number to an improper fraction; then extract the roots of the numerator and denominator for a new numerator and denominator. If the given mixed number be a surd, reduce the fractional part to a decimal, annex it to the whole number, and extract the square root from it.

22. What is the square root of $17\frac{16}{36}$? Ans. $4\frac{1}{3}$
23. What is the square root of $37\frac{25}{49}$? $6\frac{1}{7}$
24. What is the square root of $9\frac{49}{81}$? $3\frac{7}{9}$
25. What is the square root of $81\frac{64}{121}$? $9\frac{8}{11}$

SURDS.

26. What is the square root of $76\frac{14}{17}$? Ans. 8,7649+
27. What is the square root of $85\frac{14}{15}$? 9,27+
28. What is the square root of $7\frac{5}{12}$? 2,7961+

Application.

29. There is an army consisting of a certain number of men, who are placed rank and file, that is, in the form of a square, each side containing 472 men; I demand how many men the whole square contains? Ans. 222784.

30. The floor of a large house is exactly square, each side of which contains 75 feet; I want to know how many square feet are contained therein? Ans. 5625.

31. A certain general has an army of 5184 men; how many must he place in rank and file, to form them into a square? $\sqrt{5184}=72$ Ans.

To find a mean proportional between two numbers.

RULE. Multiply the two given numbers together, and extract the square root of that product.

32. What is the mean proportional between 4 and 9?

$4\times9=36$, and $\sqrt{36}=6$ Ans.

To form any body of soldiers, so that they may be double, triple, &c. as many in rank as in file.

RULE. Divide the given number of men by 2, 3, &c. according to the nature of the question, and extract the square root of the quotient, and that will be the number of men in file, which double, triple, &c. and the product will be the number in rank.

33. Suppose 13122 men be so formed, that the number in rank may be double the number in file; how many in each?

$13122 \div 2 = 6561$, $\sqrt{6561} = 81$ in file, $81 \times 2 = 162$ in rank.

EXTRACTION OF THE CUBE ROOT.

To extract the Cube Root is nothing else than to find such a number, as being first multiplied into itself, and then that product, produceth the given number, to perform which there are many rules.

RULE 1. First point your given number, beginning with the unit's place, and make a point or dot over every third figure, towards the left hand. Secondly, seek the greatest cube number in the first point, towards the left hand, putting the root thereof in the quotient, and the said cube number under the first point, and subtract it therefrom, and to the remainder bring down the next point, and call that the *resolvend.* Thirdly, triple the quotient, and place it under the resolvend, the unit's place of this under the ten's place of the resolvend, and call this the *triple quotient.* Fourthly, square the quotient, and triple the square, and place it under the triple quotient, the unit's of this under the ten's place of the triple quotient, and call this the *triple square.* Fifthly, add these two together, in the same order as they stand, and the sum will be the divisor. Sixthly, seek how often the divisor is contained in the resolvend, rejecting the unit's place of the resolvend, (as in the square root,) and put the answer in the quotient. Seventhly, cube the figure last put in the quotient, and put the unit's place thereof under the unit's place of the resolvend. Eighthly, multiply the square of the figure last put in the quotient, into the triple quotient, and place the product under the last, one place more to the left hand. Ninthly, multiply the triple square by the figure last put in the quotient, and place it under the last, one place more to the left hand. Tenthly, add the

three last numbers together, in the same order as they stand, and call that the *subtrahend*. Lastly, subtract the subtrahend from the resolvend, and if there be another point, bring it down to the remainder, and call that a new resolvend, and proceed in all respects as herein directed.

RULE 2. Take the nearest root, not too great, of the first period, for the first figure of your root, subtract its cube from said period, and to the remainder bring down the next period for a resolvend. Take three times the square of the root for a defective divisor, and seek how often it is contained in the resolvend, abating the two right-hand figures. Set down the result of this trial for the second figure of the root, and its square for the two right-hand figures of the divisor: complete the divisor by adding thereto 30 times the product of the last figure of the root into the rest. Multiply, subtract, and to the remainder bring down the next period for a new resolvend: add together the last complete divisor, the number that completed it, and twice the square of the last figure of the root, for a new defective divisor, and thus proceed throughout.

RULE 3. Point every third figure of the given cube, beginning at the unit's place, and seek the greatest cube to the first point, subtract it therefrom, put the root in the quotient, and bring down the figures in the next point to the remainder, for a resolvend. Find a divisor by multiplying the square of the quotient by 3, and see how often it is contained in the resolvend, rejecting the units and tens, and put the answer in the quotient. Cube the last figure in the quotient, and multiply all the figures in it by 3, except the last, and that product by the square of the last; then multiply the divisor by the last figure. Adding these products together gives the subtrahend, which subtract from the resolvend, and to the remainder bring down the next point, and proceed as before.

RULE 4. Point off every third figure of the given number, beginning at the unit's place; then find the nearest cube to the first point, and subtract it therefrom, put the root in the quotient, bring down the next period or point to the remainder, for a resolvend; then square the quotient, and triple the square for a divisor, and find how often it is contained in the resolvend, rejecting units and tens in the resolvend, and put the answer in the quotient. Square the last figure in the quotient, and put it on the right hand of the divisor; triple the last figure in the quotient, and multiply by the former, put it under the other, units under the tens, add them together, and multiply the sum by the last

figure in the quotient, and subtract that product from the resolvend, and bring down the figures in the next point or period, and proceed as before.

RULE 5. *By approximation.*—Find by trial a cube near to the given number, and call it the supposed cube; then say, as twice the supposed cube, added to the given number, is to twice the given number, added to the supposed cube, so is the root of the supposed cube to the true root, or an approximation to it. By taking the cube of the root thus found for the supposed cube, and repeating the operation, the root will be had to any degree of exactness.

NOTE. There are many more rules for extracting the cube root, but they are more than is contained in any one treatise extant, besides the method by logarithms.

EXAMPLES.

1. Let 314432 be a cube number, whose root is required?

	314432 (68	Ans.
First cube	216	
Resolvend	98432	
	18	triple quotient of 6.
	108	triple square of the quotient 6.
Divisor	1098	
	512	cube of 8, last figure in the quotient.
	1152	square of 8 by the triple quotient.
	864	triple square of the quotient, 6 by 8.
	98432	subtrahend.

Demonstration.—After you have pointed the given number, seek what is the greatest cube number in 314, the first point, which you will find to be 216, the nearest that is less than 314, and its root is 6, which put in the quotient, and 216 under 314, and subtract it therefrom, and there remains 98, to which bring down 432, the next period, annex it to 98, and it will make 98432 for the resolvend. Then triple the quotient 6, which make 18, write down the unit's place 8 under 3, the ten's place of the resolvend; square the quotient 6, and triple the square, and it makes 108, which write under the triple quotient, one place towards the left hand. Then add those two numbers together, which make 1098, for the divisor; seek how often the divisor is contained in the resolvend, (rejecting the unit's place thereof,) that is, how often 1098 is in 9843, which is 8 times; put 8 in the quotient, and the cube thereof below the divisor, and the unit's place under the unit's place of the resolvend. Then square the 8 last put in the quotient, and multiply 64, the square thereof, by the triple quotient 18, and the product is 1152, which set under the cube of 8, and the unit's of this under the ten's of that. Then multiply the triple square of the quotient by 8, the figure last put in the quotient, and the product is 864, which set down under the last product, a place more to the left hand. Then draw a line under these three, add them together, and the sum is 98432, which is called the *subtrahend*, and being subtracted from the resolvend, the remainder is nothing, which shews the number to be a true cubic number, whose root is 68.

Another by Rule 1.

2. Required the cube root of 5735339?

```
 .  .  .
5735339 (179   Ans.
1
----
4735
----
   3   triple of the quotient 1, the first figure.
   3   triple square of the quotient 1.
   --
  33   divisor.
  ---
  343  cube of 7, the second figure of the root.
  147  square of 7, multiplied in the triple quotient, 3.
   21  triple square of the quotient, multiplied by 7.
  ----
  3913 subtrahend.
 ------
 822339 new resolvend.
 ------
   51  triple quotient of 17, the two first figures.
  867  triple square of the quotient 17.
  ----
  8721 divisor.
  ----
   729 cube of 9, the last figure of the root.
  4131 square of 9, multiplied by the triple quotient, 51.
 7803  triple square of the quotient, 867, multiplied by 9.
 ------
 822339
 ------
```

By Rule 2.

3. Required the cube root of 470184984576?

```
                  .  .  .  .
                  470184984576(7776   Ans.
      14749       343
       1470       -----
      -----       127184
      16219       113533
     -------      --------
    1778749       13651984
      16170       12564433
    -------       --------
    1794919        1087551576
   ---------       1087551576
   181118736       ----------
      139860
   ---------
   181258596
   ---------
```

Ans. 7776, the root required.

By Rule 3.

4. What is the cube root of 99252847?

```
                                  . . .
                                  99252847(463  Ans.
                    Cube of 4 = 64
                                  ------
Square of 4=16×3=48 divisor       35252   resolvend.
                                  ------
                                    216   cube of 6.
                                   432    4 × 3 × by the square of 6.
                                  288     divisor multiplied by 6.
                                  ------
                                  33336   subtrahend.
                                  ------
Square of 46 × 3 = 6348 divisor   1916847 resolvend.
                                  ------
                                      27  cube of 3.
                                    1242  46 × 3 × by square of 3.
                                  19044   divisor multiplied by 3.
                                  ------
                                  1916847 subtrahend.
                                  ------
```

By Rule 4.

5. What is the cube root of 99252847?

```
Square of 4×3=48 divisor                  . . .
Square of 6 put to 48 equal 4836          99252847(463  Ans.
          6 × 3 × 4 =  72                 64
                       ---                ------
                       5556 × 6 =         35252
                       ---                33336
                                          ------
Square of 46 is 2116×3 = 6348 divisor     1916847  resolvend.
Square of 3 is 9 put to 6348 = 634809
          3 × 3 × 46 =    414
                          ------
                          638949 × 3 = 1916847
```

NOTE. When the quotient is 1, 2 or 3, there must be a cipher put to supply the place of tens, as above.

By Rule 5. Approximation.

6. It is required to find the cube root of 54854153?

Let 64000000 equal the supposed cube of 400. Now, 64000000×2 equal 128000000, to which add the given number, 54854153=182854153. Then the given number, multiplied by 2, is 109708306, to which add the supposed cube; thus, 109708306+64000000=173708306. Then say,

As 182854153 : 173708306 : : 400 the supposed root, to 379, the root nearly.

Again, let 54439939 be the supposed cube of 379, the supposed cube root; then 54439939×2+54854153=163734031. The given cube, 54854153×2+54439939 = 164148245. Then say,

As 163734031 : 164148245 : : 379 the supposed root, to 379,958793+ the root exactly.

Another method.

RULE 6. Divide the resolvend by 3 times the assumed root, and reserve the quotient; then subtract a twelfth part of the square of the assumed root from the quotient. Extract the square root of the remainder, and to this root add one-half of the assumed root, and the sum will be the true root, or an approximation to it. Take this approximation as the assumed root, and by repeating the process, a root farther approximated will be found, which may be repeated as often as necessary, and the root brought out to any degree of exactness.

7. What is the cube root of 436036824287?

Let 7000 equal the assumed root.
3

21000)436036824287(20763658,2994
7000 × 7000 ÷ 12 = 4083333,3333

✓ 16680324,9661

4084,15 square root.
7000÷2=3500 half the assumed root 3500

Ans. 7584,15 root very near.

If you make a second trial for more exactness, take this answer for the supposed root, and proceed as above.

8.	What is the cube root of 389017?	Ans. 73
9.	What is the cube root of 80,763?	4,32+
10.	What is the cube root of 1728?	12
11.	What is the cube root of 5735339?	179
12.	What is the cube root of 32461759?	319
13.	What is the cube root of 84604519?	439
14.	What is the cube root of 259697989?	638

When the given number consists of a whole number, and a decimal together.

RULE. Make the number of decimals to consist of 3, 6, 9, &c. places of decimals, by annexing ciphers thereto, so that there may a point fall on the unit's place of the whole number.

15. What is the cube root of 7612,812161?
Ans. 19,67+

16. What is the cube root of 93759,575070?
Ans. 45,42½

17. What is the cube root of 12000,812161?
Ans. 22,89+

18. What is the cube root of ,1218612 61?
Ans. ,425+

19. What is the cube root of ,0069761218?
Ans. ,19107+

To extract the cube root of a vulgar fraction.

RULE. Reduce the fraction to its lowest terms, and extract the cube root of the numerator and denominator for a new numerator and denominator; but if the fraction be a surd, reduce it to a decimal, and extract the root from it.

NOTE. It is better, in my opinion, always to bring a vulgar fraction to a decimal, in extracting the square, or cube root thereof.

20. What is the cube root of $\frac{54}{1728}$? Ans. $\frac{1}{4}$
21. What is the cube root of $\frac{16}{54}$? $\frac{2}{3}$
22. What is the cube root of $\frac{128}{250}$? $\frac{4}{5}$
23. What is the cube root of $\frac{375}{1331}$? $\frac{5}{8}$

SURDS.

24. What is the cube root of $\frac{4}{7}$? Ans. ,829+
25. What is the cube root of $\frac{5}{8}$? ,822+
26. What is the cube root of $\frac{2}{3}$? ,873+

To extract the cube root of mixed numbers.

RULE. Reduce the fractional part to its lowest terms, and the mixed number to an improper fraction; extract the cube root of the numerator and denominator for a new numerator and denominator; but if the mixed number given be a surd, reduce the fractional part to a decimal, annex it to the whole number, and extract the cube root therefrom.

27. What is the cube root of $578\frac{19}{27}$? Ans. $8\frac{7}{8}$
28. What is the cube root of $42\frac{21}{38}$? $3\frac{1}{2}$
29. What is the cube root of $5\frac{104}{125}$? $1\frac{4}{5}$

SURDS.

30. What is the cube root of $8\frac{5}{7}$? Ans. 2,057+
31. What is the cube root of $7\frac{3}{5}$? 1,966+
32. What is the cube root of $9\frac{1}{8}$? 2,092+

Application.

33. If a cubical piece of timber be 41 inches long, 41 inc. broad, and 41 inches deep, how many inches are contained therein? Ans. 68921.

To find two mean proportionals between two given numbers.

RULE. Divide the greater extreme by the lesser; and the cube root of the quotient, multiplied by the lesser extreme, gives the lesser mean. Multiply the said cube root by the lesser mean, and the product will be the greater mean proportional.

34. What are the two mean proportionals between 6 and 162? Ans. 18 and 54.

35. What are the two mean proportionals between 4 and 108? Ans. 12 and 36.

NOTE. If a number cannot be divided by any number less than the square root thereof, that number is a prime. All prime numbers, except 2 and 5, have 1, 3, 7 or 9, in the place of units. If the two right-hand figures of any number be divisible by 4, the whole is divisible by 4. If the three right-hand figures of any number be divisible by 8, the whole is divisible by 8. If the sum of the digits, constituting any number, be divisible by 3 or 9, the whole is divisible by 3 or 9. The product of any two numbers can have at most but as many places of figures as are in both their factors, and at least but one less. A square number cannot have more places of figures than double the places of the root, and at least but one less. A cube number cannot have more places of figures than triple the places of the root, and at least but one less.

DUODECIMALS,

OR MULTIPLICATION OF FEET, INCHES AND PARTS,

Is a Rule made use of by workmen and artificers, in casting up the contents of their work.

RULE 1. Under the multiplicand place the corresponding denominations of the multiplier. Multiply each term into the multiplicand, beginning at the lowest in the multiplicand, by the highest in the multiplier, and write the result of each under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior, on the left hand. In the same manner multiply the multiplicand by the inches or seconds in the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand. Do the same with the seconds in the multiplier, setting the result of each term two places to the right hand of those in the multiplicand.

TABLE 1st.

12 Fourths	make	-	-	-	-	-	1	Third.
12 Thirds	-	-	-	-	-	-	1	Second.
12 Seconds	-	-	-	-	-	-	1	Inch.
12 Inches	-	-	-	-	-	-	1	Foot.

RULE 2. *By Practice.*—Multiply the multiplicand by the highest denomination of the multiplier, and take parts for the remaining inferior denominations of the multiplier.

RULE 3. *By whole numbers.*—Reduce the given numbers to the lowest denomination in the question, and multiply them together, and that product divide by the several denominations, according to the table, as far as inches: then, when you have the whole amount in inches, divide by 144, and you have the answer in square feet.

RULE 4. *By Vulgar Fractions.*—Bring the inferior denominations to the fraction of a foot, and annex it to the feet in the given question, and multiply as in multiplication of vulgar fractions, and you will have the answer in feet, and the fractional part of a foot.

RULE 5. *By Decimals.*—Reduce the inferior denominations to the decimal of a foot, and annex it to the feet in the given question, and multiply as in multiplication of decimals, and the product will be the answer in feet.

TABLE 2nd.

1. Feet multiplied by feet, give feet.
2. Feet multiplied by inches, give inches.
3. Feet multiplied by seconds, give seconds, &c.
4. Inches multiplied by inches, give seconds
5. Inches multiplied by seconds, give thirds, &c.
6. Seconds multiplied by seconds, give fourths.
7. Seconds multiplied by thirds, give fifths, &c.

NOTE. In adding feet, inches, &c. place the several denominations, which are alike, under each other, and add as in compound addition, carrying 1 for every 12 to the next denomination, towards the left hand. In subtraction of feet, inches, &c. place the several denominations, which are alike, under each other, and subtract as in compound subtraction, borrowing 12 when necessary.

EXAMPLES.

1. Add 7 ft. 9 inc. 14 ft. 4 inc. 12 ft. 8 inc. and 6 ft. 6 inc. together. Ans. 41 ft. 8 inc.

2. Subtract 21 feet 9 inches from 100 feet 2 inches.
Ans. 78 feet 5 inches.

3. Let it be required to multiply 2 feet 6 inches by 2 feet 6 inches, and find the content?

By Rule 1.

	Ft.	inc.
	2	6
	2	6
	5	0
	1	3 0
Ans.	6	3 0

By Rule 2.

	Ft.	inc.
	2	6
	2	6
	5	0
6 inches ½	1	3 equal ½ the upper line.
Ans.	6	3

By Rule 3.

Ft.	inc.
2	6
12	
30 inches	
30	
144)900	
6¼ equal 6 feet 3 inches, the answer.	

Ft.	inc.
2	6
12	
30 inches	

By Rule 4.

2 ft. 6 inc. equal 2½ ft.
2 ft. 6 inc. equal 2½ ft.

$$\frac{2\frac{1}{2}}{2} \quad \frac{2\frac{1}{2}}{2}$$

$$\frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4} = 6 \text{ ft. } 3 \text{ inc.}$$

By Rule 5.

2 ft. 6 inc. equal 2½ ft. and 2½ equal 2,5
2 ft. 6 inc. equal 2½ ft. and 2½ equal 2,5

125
50

6,25 = 6¼ ft. = 6 ft. 3 inc.

	Ft.	inc.
Multiply	7	9
By	3	6
Ans.	27	1 6

	Ft.	inc.
Multiply	75	7
By	9	8
Ans.	730	7 8

	Ft.	inc.
Multiply	97	8
By	8	9
Ans.	854	7

	Ft.	inc.
Multiply	75	9
By	17	7
Ans.	1331	11 3

	Ft.	inc.
Multiply	87	5
By	35	8
Ans.	3117	10 4

	Ft.	inc.
Multiply	259	2
By	48	11
Ans.	12677	6 10

	Ft.	inc.
Multiply	179	3
By	38	10
Ans.	6960	10 6

	F.	I.
Multiply	257	9
By	39	11
Ans.	10288	6 3

	F.	I.	P.
Multiply	7	5	9
By	3	5	3
Ans.	25	8 6	2 3

	F.	I.	P.	S.	T.				
Multiply	8	4	3	5	6				
By	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		1	10	6	0	2	6		
			5	6	0	3	8	0	
				1	4	3	6	11	0
Ans.	27	7	3	5	1	8	8	11	0

A COMPLETE SYSTEM

OF

MENSURATION,

ACCORDING TO

EUCLID, EVERARD, WARD, PARDIE, F. R. S. VAN CULEN, HARRIS, AND WILLIAM HAWNEY.

MENSURATION OF SUPERFICIES.

SUPERFICIAL figures are all such as have only length and breadth, and not having any commensurable thickness.

1st. OF A SQUARE.

A square is a geometrical figure, having four equal sides, and as many right (or square) angles. To find the superficial content thereof, this is the

RULE. Multiply the side into itself, and the product is the content.

Let the side of a geometrical square be 14 yards, poles, or feet; required the superficial content?

$14 \times 14 = 196$ yards, poles, or feet, answer.

2nd. OF A PARALLELOGRAM.

A parallelogram is a figure having four sides, and as many right angles, the opposite sides thereof being equal and parallel. To find the superficial content thereof,

RULE. Multiply the length by the breadth, and the product is the superficial content.

Let a parallelogram be a long square, whose length is 18 feet, and breadth 9 feet; required the superficial content?

Ans. 162 feet.

3d. OF A RHOMBUS.

A rhombus is a figure representing a quarry of glass, having four equal sides, the opposites thereof being equal, two angles being obtuse, and two acute. To find the superficial content thereof, this is the

RULE. Multiply one of the sides by a perpendicular let fall from one of the obtuse angles, to the opposite side, and the product is the content.

Let each side of a rhombus be 15,5 feet, and the perpendicular 13,42 feet; required the superficial content?

Ans. 208,01 feet.

4th. OF A RHOMBOIDES.

A rhomboides is a figure having four sides, the opposite whereof is equal, and parallel; also four angles, the opposite whereof is equal. To find the superficial content thereof,

RULE. Multiply one of the longest sides thereof by a perpendicular let fall from one of the obtuse angles, to one of the longest sides, and the product is the content.

Let the longest sides of a rhomboides be 19,5 feet, and the perpendicular let fall be 10,2 feet; required the superficial content? Ans. 198,9 feet.

5th. OF A TRIANGLE.

A triangle is a figure having three sides and three angles. Triangles are either right-angled, or oblique-angled. Right-angled triangles are such as have one right angle; oblique-angled triangles are such as have their angles either acute or obtuse. An obtuse angle is greater than a right angle, that is, it is more than 90 degrees; and an acute angle is less than a right angle. To find the superficial content thereof, this is the

RULE. Let the triangle be of what kind soever, multiply the base by half the perpendicular, or half the base by the whole perpendicular; or multiply the whole base by the whole perpendicular, and take half the product. Any of these ways will give the content.

Let the base of a right-angled triangle be 14,1 feet, and the perpendicular 12 feet; required the content?

Ans. 84,6 feet.

Let the base of an oblique-angled triangle be 15,4 feet, and the perpendicular 7,8 feet, what is the superficial content? Ans. 60,06 feet.

To find the area of any plain triangle, by having the three sides given, without the help of a perpendicular.

RULE. Add the three sides together, and take half that sum; then subtract each side severally from that half sum, which done, multiply that half sum and the three differences continually, and out of the last product extract the square root, which square root shall be the area of the triangle sought.

Let a triangle be given, whose three sides are as follows, viz. 43,3 20,5 and 31,2 feet; required the area?

Ans. 296,31 feet.

6th. OF A TRAPEZIUM.

A trapezium is a figure having four unequal sides, and oblique angles. To find the area, or superficial content thereof, this is the

RULE. Add the two perpendiculars together, and take half the sum, and multiply that half sum by the diagonal; or multiply the whole sum by half the diagonal, and the product is the area. Or you may find the areas of the two triangles, add these two areas together, and the sum will be the area of the trapezium.

Let a trapezium be given, the diagonal whereof is 80,5 and the perpendiculars severally 30,1 and 24,5 feet; required the area? Ans. 2197,65 feet.

7th. OF IRREGULAR FIGURES.

Irregular figures are all such as have more sides than four, and the sides and angles unequal. All such figures may be divided into as many triangles as there are sides, wanting two. To find the area of such figures, they must be divided into trapeziums and triangles, by lines drawn from one angle to another, and to find the areas of the trapeziums and triangles severally, and then add all the areas together, and you will have the area of the whole figure.

NOTE. This figure being composed of triangles and trapeziums, and these being shewn in the 5th and 6th problems aforegoing, it will be needless to mention any thing more in this place.

8th. OF REGULAR POLYGONS.

Regular polygons are all such figures as have more than four sides, all the sides and angles thereof being equal.

Polygons are denominated from the number of their sides and angles.. To find the area, or superficial content of any regular polygon, this is the

RULE. Multiply the whole perimeter, or sum of the sides, by half the perpendicular let fall from the centre to the middle of one of the sides; or multiply the half perimeter by the whole perpendicular, and the product is the area.

If the figure consists of

5 equal sides and angles, it is called a regular Pentagon.
6 equal sides and angles, it is called a regular Hexagon.
7 equal sides and angles, it is called a regular Heptagon.
8 equal sides and angles, it is called a regular Octagon.
9 equal sides and angles, it is called a regular Enneagon.
10 equal sides and angles, it is called a regular Decagon.
11 equal sides and angles, it is called a regular Endecagon.
12 equal sides and angles, it is called a regular Dodecagon.

Let a regular hexagon be given, whose sides are severally 14,6 feet, and the sum of all the sides 87,6 feet, and the perpendicular 12,64 feet: required the area, or superficial content? Ans. 553,632 feet.

A Table for the more ready finding the area of a polygon.

RULE. Multiply the square of the side by the tabular number, and the product is the area of the polygon.

No. of sides.	Names.	Multipliers.
3	Trigon	,433013
4	Tetragon	1,000000
5	Pentagon	1,720477
6	Hexagon	2,598076
7	Heptagon	3,633959
8	Octagon	4,828427
9	Enneagon	6,181827
10	Decagon	7,694209
11	Endecagon	8,514250
12	Dodecagon	9,370125

How to find these tabular numbers.—These numbers are found by trigonometry, thus: Find the angle at the centre of the polygon, by dividing 360 degrees by the number of sides of the polygon.

Let each side of a dodecagon be 1; required the area thereof? Ans. 9,330125.

9th. OF A CIRCLE.

A circle is a plain figure, contained under one line, which is called a circumference, unto which all lines, drawn from

a point in the middle of the figure, called the centre, and falling upon the circumference thereof, are all equal the one to the other. The circle contains more space than any plain figure of equal compass.

Demonstration.—Every circle may be conceived to be a polygon of an infinite number of sides; and the semidiameter must be equal to the perpendicular of such a polygon, and the circumference of the circle equal to the periphery of the polygon: therefore half the circumference, multiplied by half the diameter, gives the area.

F. Ignat and G. Pardie says—"Every circle is equal to a rectangle triangle, one of whose legs is the radius, and the other a right line, equal to the circumference of the circle; for such a triangle will be greater than any polygon inscribed, and less than any polygon circumscribed, by the 24th, 25th, 26th and 27th articles of the fourth book of his Elements of Geometry, and therefore must be equal to the circle; for should it be greater than the circle, be the excess as little as it will, a polygon may be circumscribed, whose difference from the circle shall be yet less than the difference between that circle and the rectangle triangle; and that that polygon will be less than the triangle, is absurd. If it be said that this rectangled triangle is less than the circle, an inscribed polygon may be made, which shall be greater than that triangle, which is impossible.

"This cannot but be admitted as a principle—that if two determinate quantities are such, that if every imaginable quantity, which is greater or less than one, is also greater or less than the other, these two quantities must be equal. And this principle being granted, which is in a manner self-evident, it may directly be proved, that the triangle before mentioned is equal to the circle, because every imaginable inscribed figure, which is less than the circle, is also less than the triangle; and every circumscribed figure, greater than the circle, is also greater than the triangle.

"The proportion of the diameter of a circle to the circumference was never yet exactly found out, notwithstanding many eminent learned men have laboured very far therein, among whom the excellent Van Culen hath hitherto outdone all, in his having calculated the said proportion to 36 places of decimals, which are engraven on his tombstone in St. Peter's church, in Leyden, which numbers are these:

Diameter,

1,00000,00000,00000,00000,00000,00000,00000.

Circumference,

3,14159,26535,89793,23846,26433,83279,50288.

"Of which large number, these six places, 3,14159, answering to the diameter, 1,00000, may be sufficient."

I will give many proportions, but for the most part that of Van Culen, as being most exact.

If the diameter of any circle

be	× or ÷	by	3,14159	the product	is the circumference.
			,31831	the quotient	
be	× or ÷	by	,886227	the product	is the side of an equal square.
			1,128379	the quotient	
be	× or ÷	by	,707016	the product	is the side of the square inscribed.
			1,414213	the quotient	

If the square of the diameter of any circle

be { × / ÷ } by { ,785398 / 1,273241 } { the product / the quotient } is the area.

If the circumference of any circle

be { × / ÷ } by { ,31831 / 3,14159 } { the product / the quotient } is the diameter.

If the circumference of any circle

be { × / ÷ } by { ,282094 / 3,544907 } { the product / the quotient } is the side of the square equal.

If the circumference of any circle

be { × / ÷ } by { ,2756646 / 3,6275939 } { the product / the quotient } is the side of the equilateral trian. inscribed.

If the circumference of any circle

be { × / ÷ } by { ,225079 / 4,442877 } { the product / the quotient } is the side of the square inscribed.

If the square of the circumference of any circle

be { × / ÷ } by { ,079577525 / 12,56636217 } { the product / the quotient } is the area.

If the area of any circle

be { × / ÷ } by { 1,273241 / ,785398 } { the product / the quotient } is the square of the diameter.

If the area of any circle

be { × / ÷ } by { 12,56636217 / ,079577525 } { the product / the quotient } is the square of the circumference.

When the diameter of a circle is 1, and the diameter of any other is 2, the circumference of the first circle is equal to the area of the second, 3,141592. If the circumference be 4, the diameter and area equal 1,273241. If the diameter be 4, the circumference and area equal 12,566368. If the diameter of one circle be double to that of another, the area of the first circle will be 4 times the area of the second. If the circumference of a circle be 1, the area will be ,079577. If the area be required, when the circumference is 1, first find what the diameter will be; thus,

cir. *diam.* *cir.*
As 3,1416 : 1 :: 1 : ,318309 which is the diameter, when the circumference is 1. Then multiply half ,318309 by half 1, that is, ,159154 by ,5 and the product is ,079577 which is the area of a circle, whose circumference is 1.

If the area of any circle be given to find the side of a square equal, you need but extract the square root of the area given, and it is done. So the square root of ,7854 is ,88,62 which is the side of a square equal, when the diameter is 1. And if you extract the square root of ,079577 it will be ,2821 which is the side of the square, equal to the circle whose circumference is 1.

Again, if the side of a square within a circle be required, if you square the semidiameter, and double that square, and out of that sum extract the square

root, that shall be the side of the square which may be inscribed in that circle. So if the diameter of a circle be 1, then the half is ,5 which squared is ,25 and this doubled is 5, whose square root is ,7071 the side of the square inscribed.

Let it be required to find the area of a circle, whose diameter is an unit.

By the proportion of Van Culen, if the diameter be 1, the circumference will be 3,1415926, &c. Then multiply half the circumference by half the diameter, and the product is the area.

EXAMPLES.

Problem 1. Having the diameter and circumference to find the area.

Rule. Every circle is equal to a parallelogram, whose length is equal to half the circumference, and the breadth equal to half the diameter; therefore multiply half the circumference by half the diameter, and the product is the area of the circle.

1. Let the diameter of a circle be 22,6 and the circumference 71, what is the area of said circle? Ans. 401,15.

Problem 2. Having the diameter of a circle to find the circumference.

2. Let the diameter of a circle be 22,6 feet, required the circumference thereof? Ans. 71+ feet.

Problem 3. Having the circumference of a circle to find the diameter.

3. Let the circumference of a circle be 71 feet, required the diameter? Ans. 22,6 feet.

Problem 4. Having the diameter of a circle to find the area. All circles are in proportion, one to another, as are the squares of their diameters, (by Euclid, 12, 2.)

In the Key I will give Van Culen's proportion, and also the proportions of Metius and Archimedes.

4. Let the diameter of a circle be 22,6 feet, required the area?

According to Van Culen 401,15 feet.
Metius 401,15 feet. } Ans.
Archimedes 401,31 feet.

Problem 5. Having the circumference of a circle to find the area.

Demonstration by Van Culen.—Because the diameters of circles are proportional to their circumferences, that is, as the diameter of one circle is to its circumference, so is the diameter of another circle to its circumference; therefore the areas of circles are to one another as the squares of their circumferences.

5. Let the circumference of a circle be 71 feet, required the area? Ans. 401,15 feet.

Problem 6. By having the diameter to find the side of a square equal thereunto, that is, equal in area to that circle.

6. Let the diameter of a circle be 22,6 feet, required the side of a square equal in area to the circle, of which the diameter is here given? Ans. 20,02872 side.

Problem 7. By having the circumference to find the side of the square equal.

7. Let the circumference of a circle be 71 feet, required the side of a square which will be equal in area?
Ans. 20,0291 feet.

Problem 8. Having the diameter to find the side of a square which may be inscribed in that circle.

8. Let the diameter of a circle be 22,6 feet, required the side of a square equal, when inscribed in the circle, of which the diameter is here given? Ans. 15,98046 feet.

Problem 9. Having the circumference to find the side of a square which may be inscribed.

9. Let the circumference of a circle be 71 feet, required the side of an inscribed square equal?
Ans. 15,9821 feet.

Problem 10. Having the area to find the diameter.

10. Let the area of a circle be 401,15 feet, required the diameter of that circle? Ans. 22,599 diameter.

Problem 11. Having the area to find the circumference.

11. Let the area of a circle be 401,15 feet, required the circumference of that circle? Ans. 70,999+ feet.

Problem 12. Having the area to find the side of a square inscribed.

12. Let the area of a circle be 401,15 feet, required the side of a square inscribed, which will be equal in area?
Ans. 15,98 feet.

Problem 13. Having the side of a square to find the diameter of the circumscribing circle.

13. Let the side of a square be 15,98 feet, required the diameter of a circle which shall circumscribe that given square? Ans. 22,598916 feet.

Problem 14. Having the side of a square to find the diameter of a circle equal.

14. Let the side of a square be 20,0291 feet, required the diameter of a circle equal in area to that square?
Ans. 22,5928248 feet.

Problem 15. Having the side of a square to find the circumference of a circumscribing figure.

15. Let the side of a square be 15,98 feet, required the circumference of a circle that will encompass that square.
Ans. 70,999+ cir.

Problem 16. Having the side of a square to find the circumference of a circle that will be equal thereunto.

16. Let the side of a square be 20,0291 feet, required the circumference that will be equal in area to that given square? Ans. 71,0051595 feet cir.

In several of the foregoing problems, where the diameter and circumference are required, the answers are not exactly the same as the diameter and circumference of the given circle, but are sometimes too much, and sometimes too little, as in the two last problems, where the answers in each should be 71, the one being too much, and the other too little. The reason of this is, the small defect that happens to be in the decimal fraction, they being sometimes too great, and sometimes too little; yet the defect is so small, that it is needless to calculate them to more exactness.

10th. OF A SEMICIRCLE.

To find the area of a semicircle, this is the

RULE. Multiply the fourth part of the circumference of the whole circle, (that is, half the arch line,) by the semidiameter, and the product is the area.

NOTE. The semidiameter, or radius, is a straight line, drawn off the centre of any circle to the circumference of the same.

Let the diameter of a semicircle be 22,6 feet, and the circumference 35,5 feet; required the area of the semicircle? Ans. 200,575 feet.

If only the diameter of the semicircle be given, you may say, by the Rule of Three, as 1 is to ,3927 so is the square of the diameter to the area.

11th. OF A QUADRANT.

To find the area of a quadrant, or fourth part of a circle, this is the

RULE. Multiply half the arch line of the quadrant, (that is, the eighth part of the circumference of the whole circle,) by the semidiameter, and the product is the area of the quadrant.

Required the area of a quadrant, whose semidiameter is 11,3 and the arch line 17,75 feet? Ans. 100,2875 area.

These are the rules and ways commonly given for finding the area of a semicircle and quadrant; but I think it is as good a way to find the area of the whole circle, and then take half that area for the semicircle, and a fourth part for the quadrant. Before I proceed to shew how to find the area of the sector, and segment of a circle, I shall shew how to find the length of the arch line arithmetically.

To find the length of the arch line arithmetically.

RULE. Multiply the chord of half the segment by 8, and from the product subtract the chord of the whole segment, and divide the remainder by 3, and the quotient is the arch line sought.

	19,8	= ½ the chord of the segment.
	8	
	158,4	
The whole chord	34,4	
	3)124,0	
Ans.	41,333	the arch line.

How to find the diameter of a circle by having the chord and versed sine of the segment arithmetically.

Let a segment be given, whose chord is 36, and the versed sine 6; required the diameter of that circle?

	18	= ½ the chord.
	18	
	6)324	
	54	the part wanting.
	6	the versed sine.
Ans.	60	the diameter of the circle.

12th. OF THE SECTOR OF A CIRCLE.

A sector of a circle is comprehended under two radii, or semidiameters, which are supposed not to make one right

line, and a part of the circumference, whence a sector may be either less or greater than a semicircle. To find the area, or superficial content thereof, this is the

RULE. Multiply half the arch line by the semidiameter, and the product is the area.

Let the sector of a circle be given, whose semidiameter is 24,5 feet, and the arch line 45,6 feet; required the area of the sector? Ans. 558,6 feet.

13th. OF THE SEGMENT OF A CIRCLE.

A segment of a circle is a part terminating by a right line, less than the diameter, called a chord, and by a part of the circumference. To find the area of the segment of a circle, this is the

RULE. Find the semidiameter of the circle by what has been shewn, and the arch line; then multiply half the arch line by the semidiameter, so you have the area of the sector. Then subtract the versed sine from the semidiameter, the remainder is the perpendicular of the triangle, and multiply the half chord by the perpendicular, and the product is the area of the triangle. Then subtract the area of the triangle from the area of the sector, and the remainder is the area of the segment.

Let it be required to find the area of the segment of a circle, whose chord line is 35, and the versed sine 9,6 feet. Ans. 236,060 the area.

14th. OF COMPOUND FIGURES.

Mixed, or compound figures, are such as are composed of rectilineal and curvilineal figures together.

To find the area of such figures, you must find the area of the several figures, of which the whole compound figure is composed, and add all the areas together, and the sum will be the area of the whole compound figure.

NOTE. As those compound figures are best understood by having the figure of such before the learner, and as there are none in this treatise, I will not say any thing more concerning them, but this, that the area of such figures is found in the same manner as a trapezium.

15th. OF AN ELLIPSIS.

An ellipsis, or oval, is a figure bounded by a regular curve line returning into itself, but of its two diameters, cutting each other in the centre; one is longer than the other, in which it differs from the circle. To find the area thereof,

RULE. Multiply the transverse diameter by the conjugate, and multiply that product by ,7854. This last product is the area of the oval.

Let it be required to find the area of an ellipsis, whose transverse diameter is 61,6 feet, and conjugate diameter 44,4? Ans. 2148,100416 area.

16th. OF A PARABOLA.

A parabola is a curvilineal figure, made by the section of a cone, being cut by a plane parallel to one of its sides.

Every parabola is two-thirds of its circumscribing parallelogram; therefore, to find the area thereof, this is the

RULE. Multiply the base, or greatest ordinate, by the perpendicular height, and multiply that product by 2, and divide the last product by 3, and the quotient will be the area of the parabola.

Let it be required to find the area of a parabola, whose base is 53,75 feet, and the perpendicular 39,25?
Ans. 1406,4583 area.

MENSURATION OF SOLIDS.

Solid bodies are such as do consist of length, breadth and thickness, as stone, timber, globes, bullets, &c.

1st. OF A CUBE.

A cube is a square solid, comprehended under six geometrical squares, being in the form of a dye. To find the solid content, this is the

RULE. Multiply the side of the cube into itself, and that product again by the side: the last product will be the solidity, or solid content of the cube.

Let it be required to find the solid contents of a cubic stone, each side thereof measuring 17,5 inches?
Ans. 5359,375 inches = 3,175.

2nd. OF A PARALLELOPIPEDON.

A parallelopipedon, or square prism, representing a square piece of timber or stone, each side of its square base being 21 inches, and its length 15 feet; required the solid content thereof?

RULE. Find the area of the base, and multiply said area by the length in inches, and divide that product by 1728; or multiply the area of the base by length in feet, and divide by 144, and you will have the solid content in feet.

Ans. 45,9+ feet, the solidity.

3d. OF A TRIANGULAR PRISM.

A prism is a solid contained under several planes, and having its bases like, equal, and parallel.

The solid content of a prism, (whether triangular or multangular,) is found by multiplying the area of base into the length or height, and the product is the solid content.

Let it be required to find the solid content of a triangular prism, whose base is 15,6 inches, each side and the perpendicular 13,51 inches, and the length of the solid 19,5 feet?

Ans. 14,27 feet.

4th. OF A PYRAMID.

A pyramid is a solid figure, whose base is a polygon, and whose sides are plain triangles, their several tops meeting together in one point. To find the solid content thereof,

RULE. Multiply the area of the base by a third part of the altitude, or length, and the product is the solid content of the pyramid.

Let each side of the base of a square pyramid be 18,5 inches, and the perpendicular height 15 feet; required the solidity of that pyramid? Ans. 11,88 feet.

NOTE. Every pyramid is a third part of the prism, that hath the same base and height, (by Euclid, 12, 7.)

5th. OF A CYLINDER.

A cylinder is a round solid, having its bases circular, equal, and parallel, in form of a rollingstone. To find the solid content thereof, this is the

RULE. Multiply the area of the base by the length, and the product is the solid content.

Let it be required to find the solid content of a rollingstone, whose diameter is 21,5 inches, and the length is 16 feet? Ans. 40,34 feet.

6th. OF A CONE.

A cone is a solid, having a circular base, and growing smaller and smaller, till it ends in a point, which is called

the vertex, and may be nearly represented by a sugar-loaf. To find the solidity thereof, this is the

RULE. Multiply the area of the base by a third part of the perpendicular height, the product is the solid content.

Let it be required to find the solid content of a cone, whose diameter of the base is 26,5 inches, and the height of the cone 16,5 inches? Ans. 21,06 feet.

7th. OF THE FRUSTUM OF A PYRAMID.

A frustum of a pyramid is the remaining part, when the top is cut off by a plane parallel to the base. To find the solid content thereof, this is the

RULE. To the rectangle (or product) of the sides of the two bases, add the sum of their squares, and that sum being multiplied into one-third part of the frustum's height, will give its solidity, if the base be square.

Or multiply the areas of the two bases together, and to the square root thereof add the two areas, and that sum multiplied by one-third of the height, gives the solidity of any frustum, square, or multangular.

Let it be required to find the solid content of the frustum of a square pyramid, whose side of the greater base is 18 inches, and the side of the lesser 12 inches, and the height 18 feet? Ans. 28,5 feet.

8th. OF THE FRUSTUM OF A CONE.

A frustum of a cone is that part which remains when the top-end is cut off by a plane parallel to the base. To find the solid content, the rules are the same in effect as for the frustum of a pyramid.

RULE. To the rectangle of the diameter of the two bases, add the squares of the said diameters, and multiply the sum of ,7854; the product will be the triple of a mean area, which multiplied by one-third of the perpendicular height, the product will be the solid content.

Let the solid content of the frustum of a cone be required, whose greater diameter is 18 inches, and the lesser diameter 9 inches, and the length 14,25 feet? Ans. 14,6775 feet.

9th. TO MEASURE THE FRUSTUM OF A RECTANGLED PYRAMID, CALLED A PRISMOID,

Whose bases are parallel one to another, but disproportional, this is the

RULE. To the greater length add half the lesser length, and multiply the sum by the breadth of the greater base, and reserve the product; then to the lesser length add half the greater length, and multiply the sum by the breadth, or the lesser base, and add this product to the reserved, and multiply that sum by a third part of the height, and the product is the solid content.

Let the length of the greater base of a prismoid be 38 inches, and its breadth 16 inc. and the length of the lesser base 30 inches, and its breadth 12 inches, and the height 6 feet; required the solid content?

Ans. 19,94 feet, the content.

10th. TO MEASURE A CYLINDROID,

That is, a frustum of a cone, having its bases parallel to each other, but unlike.

RULE. To the longest diameter of the greater base, add half the longest diameter of the lesser base, and multiply the sum by the shortest diameter of the greater base, and reserve the product. Then to the longest diameter of the lesser base, add half the longest diameter of the greater base, and multiply the sum by the shortest diameter of the lesser base, and add the product of the former reserved sum, and that sum will be the triple square of a mean diameter, which multiplied by ,7854, and the product multiplied by a third part of the height, the product is the solid content.

Let it be required to find the solid content of a cylindroid, whose bottom base is an oval, the transverse diameter being 44 inches, and the conjugate diameter 14 inches; and the upper base is a circle, whose diameter is 26 inches, and the height of the frustum 9 feet?

Ans. 33,47 feet, the content.

11th. OF A SPHERE OR GLOBE.

A sphere or globe is a round solid body, every part of whose surface is equally from a point within it, called the centre; and it may be conceived to be formed by the revolution of a semicircle round its diameter. To find its solidity, this is the

RULE. Multiply the axis or diameter into the circumference, and the product is the superficial content, which multiplied by a sixth part of the axis, the product is the solidity.

Or, as 21 is to 11, so is the cube of the axis to the solid content. Or, as 1 is to ,5236, so is the cube of the axis to the solid content.

Let the solid content of a globe be required, whose axis is 20 inches? Ans. 4188,8 inches.

NOTE. If the axis of a globe be 1, the solidity will be ,5236; and if the circumference be 1, the solidity will be ,016887. The superficies of every globe is equal to four times the area of its greatest circle. The solidity of every globe is two-thirds of its circumscribing cylinder.

12th. OF A SPHEROID.

A spheroid is a solid, resembling an egg. To find the solid content thereof, this is the

RULE. Multiply the square of the diameter of the greatest circle by the length, and the product multiply again by ,5236. This last product will be the solidity of the spheroid.

Let the solidity of a spheroid be required, the diameter of whose greatest circle is 33 inches, and length 55 inches? Ans. 31361,0220 inches, the solidity.

13th. OF A PARABOLIC CONOID.

A parabolic conoid is something like a half spheroid, having its sides somewhat straiter. It is generated by supposing a semiparabola turned about its axis. To find the solid content thereof, this is the

RULE. Multiply the square of the diameter of its base by ,7854, and multiply that product by half the height, and the last product shall be the solid content.

Let the solidity of a parabolic conoid be required, the diameter of whose base is 36 inches, and its height 33 inc.? Ans. 9,71 feet, the content.

14th. OF A PARABOLIC SPINDLE.

If an acute parabola be supposed to be moved about its greatest ordinate, it will form a solid, called a parabolic spindle. To find the solid content, this is the

RULE. Multiply the square of the diameter of its greatest circle, 41888, (being three-fifths of ,7854,) and that product by its length, and the last product is the solid content.

Let it be required to find the solid content of a parabolic spindle, whose greatest diameter is 36 inches, and its length 99 inches? Ans. 31,10184 feet.

Of measuring the works of the several artificers, relating to building, and what methods and customs are observed therein.

1st. OF CARPENTERS' WORK.

The carpenter's works, which are measurable, are flooring, partitioning, and roofing, all which are measured by the square of 10 feet long, and 10 feet broad, so that the square contains 100 square feet.

1st. OF FLOORING.

1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad, how many squares of flooring are there in that room?

	Ft.	*ins.*
Multiply	57	3
By	28	6
1,00)16,31	7	6

Ans. 16,31 16 squares 31 feet.

NOTE. Any parts under a foot are not counted.

2. Let a floor be 53 feet 6 inches long, and 47 ft. 9 ins. broad, how many squares are contained in that floor?

Ans. 25 squares 54 feet.

2nd. OF PARTITIONING.

3. If a partition between rooms be in length 82 feet 6 inches, and in height 12 feet 3 inches, how many squares are contained therein? Ans. 10 squares 10 feet.

3d. OF ROOFING.

It is a rule among workmen, that the flat of any house, and half the flat thereof, taken within the walls, is equal to the measure of the roof of the same house; but this is when the roof is true pitched—for if the roof be more flat or steep than the true pitch, it will measure more or less accordingly.

4. If a house within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad, how many squares of roofing will cover that house? Ans. 12 squares 18 feet.

NOTE. There are other works about a building, done by the carpenter, which are measured by the foot, running measure, that is, by the number of feet in length only, as cornices, doors and cases, window-frames, guttering, lintels, skirtboards, &c. In the measuring of flooring, after you have measured the whole floor, you must deduct out of it the well-holes for the stairs and chimnies; and in partitioning, for the doors, windows, &c. except (by

agreement) they are to be included. In measuring of roofing, seldom any reductions are made for the holes for the chimney-shafts, the vacancies for Lutheren lights and skylights, for they are more trouble to the workman, than the stuff which would cover them is worth.

2nd. OF BRICKLAYERS' WORK.

The principal is walling and chimney-work.

1st. OF WALLING.

Bricklayers commonly measure their work by the rod square of 16½ feet, so that one rod in length, and one in breadth, contain 272,25 square feet; but in some places the custom is to allow 18 feet to the rod, that is, 324 square feet. In some places, the usual way is to measure by the rod of 21 feet long, and 3 feet high, that is, 63 square feet, and here they never regard the thickness of the wall; but the usual way is to moderate the price according to the thickness.

Commonly brick walls, that are measured by the rod, are to be reduced to a standard thickness of a brick and a half thick, (if not agreed on the contrary;) and to reduce a wall to standard thickness, this is the

RULE. Multiply the number of superficial feet, that are found to be contained in any wall, by the number of half-bricks that wall is in thickness, and one-third part of that product shall be the content thereof in feet, reduced to the standard thickness of one brick and a half.

1. If a wall be 72 feet 6 inches long, and 19 feet 3 inches high, and 5½ bricks thick, how many rods of brickwork are contained in that wall, when reduced to standard thickness?

Ans. 18 rods 3 qrs. 12 feet.

NOTE. In reducing feet into rods, they usually reject the fractional parts, (,25) and divide by 272 instead of 272,25 square feet, a thing very insignificant.

2. If a wall be 245 feet 9 inches long, 16 feet 6 inches high, and 2½ bricks thick, I demand how many rods of brickwork are contained in that wall, when reduced to standard thickness?

Ans. 24 rods 3 qrs. 24 feet.

To find proper divisors.

Divide 3, (the number of half-bricks in 1½,) by the number of half-bricks in the thickness, which will be a divisor that will give the answer in feet: but if you would have a divisor to bring the answer in rods at once, multiply 272,25 by the divisor found for feet, and the product will be a divisor which will give the answer in rods.

Let it be required to find a divisor proper to reduce a wall of 3 bricks thick, to a standard thickness of 1½ bricks?

3 equal the number of half-bricks in 1½ bricks thick.
6 equal the number of half-bricks in the given wall.

Then 3 ÷ 6 = ,5 which is a divisor that will give the answer in feet. Then multiply 272,25 by ,5 and the product is 136,125, the divisor, which will give the answer in rods, that is, as 136,125 is to the length of the wall, so is the height to the content in rods. Or, as ,5 is to the length, so is the height to the content in feet. After the same manner you may find divisors for any other thickness, which you will find to be as expressed in the following table.

The thickness of the wall.	Divisors for the answer in feet.	Divisors for bringing the answer in rods.
1 brick thick	1,5	408,375
1½ bricks thick	1,	272,25
2 bricks thick	,75	204,1875
2½ bricks thick	,6	163,35
3 bricks thick	,5	136,125
3½ bricks thick	,4285	116,659
4 bricks thick	,375	102,0937

To find how many bricks are necessary to build a house of given dimensions.

RULE. From the whole circumference of the wall, measured round on the outside, subtract four times the thickness; then multiply the remainder by the height, and that product by the thickness of the wall gives the solid content of the whole wall, which multiplied by the number of bricks contained in a solid foot, gives the answer.

EXAMPLE.

How many bricks, 8 inches long, 4 inches broad, and 2½ inches thick, will build a house 44 feet long, 40 feet wide, and 20 feet high, the wall to be a foot thick?

Ans. 70848 bricks.

2nd. OF CHIMNIES.

If you are to measure a chimney standing alone by itself, without any party-wall being adjoined, then girt it about for the length, and the height of the story is the breadth: the thickness must be the same as the jambs are of, provided that the chimney be wrought upright, from the mantel-tree to the ceiling, not deducting any thing for the vacancy between the floor, or hearth; and the mantel-tree, because of the gatherings of the breast and wings, to make room for the hearth in the next story.

If the chimney-back be a party-wall, and the wall be measured by itself, then you must measure the depth of the two jambs, and the length of the breast for a length, and the height of the story for the breadth, at the same thickness your jambs were of.

When you measure chimney-shafts, (that part of the chimney above the roof,) girt them with a line round about the least place of them, for the length, and the height shall be your breadth. If they be 4 inch work, you must set down their thickness at one brickwork; but if they be wrought 9 inches thick, (as sometimes they are, when they stand high, and alone, above the roof,) you must account your thickness a brick and a half, in consideration of widths and pargetting, and trouble of scaffolding. It is customary in most places to allow double measure for chimnies.

Suppose you would find how many rods of brickwork are in a chimney three stories high, according to double measurement, which hath a double tunnel towards the top, and a double shaft. I first begin by giving you the dimensions.

Of the first story.—The breastwall, and the two angles together, are 18 feet 9 inches, and the height of the square is 12 feet 6 inches.

Second story.—The length of the breastwall and two angles are 14 feet 6 inches, and the height 9 feet.

Third story.—The height is 7 feet, and the length of the breastwall and two angles are 10 feet 5 inches.

Shafts.—The compass of the chimney-shafts is 18 feet 9 inches, and the height 6 feet 6 inches.

Fetters.—The depth of the middle fetter, that parts the funnels, is 12 feet, and its wideness 1 foot 3 inches.

RULE. Find the content of each story separately, the fetter and shafts, and add the five products together; then double that sum, and you have the content in feet, which bring to rods by dividing 272,25, or 272, rejecting ,25 into the double sum, (as the work is allowed to be 1½ brickwork,) and you will have the answer, which is 3 rods 3 qrs. 62 feet.

NOTE. This is all the measure that can be allowed, when the chimney stands in a gavel or sidewall, in which case the back of the chimney, (here not measured,) is accounted part of the gavel: but if the chimnies stand by themselves, as all stacks of chimnies do, in such case it is all chimney-work, and therefore ought to be measured double on all sides.

3d. OF PLASTERERS' WORK.

The plasterer's works are principally of two kinds, viz. First, works lathed or plastered, which they call *ceilings*; second, works rendered, which are of two kinds, viz. upon

brick walls, or between quarters, in the partition between rooms; all which are measured by the yard square, or square of 3 feet, which is 9 feet.

1st. OF CEILING.

If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches broad, how many yards doth that ceiling contain?

Ans. 162,65 yards.

2nd. OF RENDERING.

If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high, how many yards are in those partitions? Ans. 176,87 yards.

NOTE. Painting is measured by the yard, if not agreed on otherwise.

4th. OF JOINERS' WORK.

Joiners measure their work by the yard square; and in taking the height of any room, where there is a cornice about, and swelling panels, and mouldings, they with a string begin at the top, and girt over all the mouldings, which make the room to measure much higher than it is. For measuring about the room, they only take it as it is upon the floor.

If a room or wainscoting girt downwards, over the mouldings, be 15 feet 9 inches high, and 126 feet 3 inches in compass, how many yards doth it contain?

Ans. 220 yards 8 feet.

NOTE. I think it unnecessary to introduce painter's or glazier's work in this treatise, as they are of no use.

5th. OF MASONS' WORK.

Masons measure their work sometimes by the foot solid, sometimes by the foot superficial, but generally by the rod, that is, 16 ft. 6 inc. long, 1 foot high, and 1 ft. 6 inc. thick.

EXAMPLES.

1. If a wall be 97 feet 5 inches long, 18 feet 3 inches high, and 2 feet 3 inches thick, how many solid feet are contained in that wall? Ans. 4000 feet 2 inches.

2. If a wall be 107 feet 9 inches long, and 20 feet 6 inches high, how many superficial feet are contained in that wall?

Ans. 2208,875 feet.

3. If a wall be 40 feet long, 20 feet high, and 2 feet thick, how many rods of mason-work are contained in said wall?

$$16,6 \times 1,6 = 24,9 = 24,75$$
$$40 \times 20 \times 2 = 1600 \div 24,75 = 64,64 \text{ rods, answer.}$$

MEASURING OF BOARDS AND TIMBER.

1st. OF BOARD MEASURE.

To measure a board is no other than to measure a long square.

1. If a board be 16 inches broad, and 13 feet long, how many feet are contained therein? Ans. 17 ft. 4 inc.

2nd. OF SQUARED TIMBER.

By squared timber is here meant such as have equal bases, and the sides straight and parallel. The rules for measuring all such solids are shewn in the measuring of a parallelopipedon, page 174.

2. If a piece of timber be 15 inches square, and 18 feet long, how many solid feet are contained therein?
Ans. 28 ft. 1 inc. 6 p.

3. If a piece of timber be 2 feet 9 inches deep, 1 foot 7 inches broad, and 16 feet 9 inches long, how many feet of timber are in that piece? Ans. 72 ft. 11 inc.

Multiply the depth, breadth and length together, and the product will be the content.

Error.—A common error is committed, for want of art, in measuring these last sorts of solids. Many add the depth and breadth together, and take half of that sum for the side of a mean square. This error, though it is but small, when the depth and breadth are pretty near equal, yet if the difference between the depth and breadth be great, the error is very considerable; for the piece of timber thus measured will be more than the truth, by a piece whose length is equal to the length of the piece of timber to be measured, and the square equal to half the difference of the breadth and depth.

The answer to the above question, in the false way, is 78,62 feet.

I will shew both ways in the Key.

To find how much in length makes a foot of any squared timber.

RULE. The following directions are general for all timber, which is of equal thickness from end to end, whether it be square, triangular, multangular, or round.

Always divide 1728, (the solid inches in a foot,) by the area of the base, and the quotient is the length of a foot.

4. If a piece of timber be 18 inches square, how much in length will make a foot solid? Ans. 5¼ inches.

5. If a piece of timber be 22 inches deep, and 15 inches broad, how much in length will make a foot?
Ans. 5,23 inches.

3d. OF UNEQUAL SQUARED TIMBER.

By unequal squared timber is meant all such as have unequal bases, that is, such as is thicker at one end than at the other, as most timber trees are when they are hewn, and brought to their squares. The usual way to measure such timber, is to take a square about the middle of the piece, which is taken for a mean square. This way, when the piece is pretty near as thick at one end as at the other, is something near the truth; but when there is a great disproportion between the ends of the piece, the error is considerable. All such solids being the frustums of pyramids, the true way of measuring them is by the rules laid down for frustums of pyramids. I will give the answer both by the true and false way.

6. If a piece of timber be 25 inches square at the greater end, and 9 inches square at the lesser end, and 20 feet long, how many feet of timber are in that tree?

Ans. 40,13 feet by the false way.
48,101 feet by the true way.

7. If a piece of timber be 32 inches broad, and 20 inches deep at the greater end, and 10 inches broad and 6 deep at the lesser end, and 18 feet long, how many feet of timber are in that piece?

Ans. Content the true way, 37,33 feet.
Content the false way, 34,12 feet.

4th. OF ROUND TIMBER, WHOSE BASES ARE EQUAL.

The usual way to measure round timber trees, is to girt them about the middle with a string, and take the fourth part of that girth for the side of a square, by which they measure the piece of timber, as if it was square.

That this method is an error, I shall make appear in the following demonstration: If the circumference of a circle be 1, the area will be ,07958; then the fourth part of 1 is ,25 which squared is ,0625. This they take for a mean area, instead of ,07958: therefore the true content always bears such proportion to the content found by the above customary false way, as ,07958 is to ,0625 which is nearly as 23 to 18; so that in measuring by the customary false way, there is above the one-fifth part lost of what the true content ought to be. This error, though it has been in practice for a series of years, ought to be omitted, if they follow the correct method, as is here laid down.

8. If a piece of timber be 96 inches in circumference, or girth, and 18 feet long, how many feet of timber are contained therein?

Ans. Content the false way, 72 feet.
Content the true way, 91,67 feet.

9. If a piece of timber be 86 inches girth, and 20 feet long, how many feet are contained therein?

Ans. Content the false way, 64,2 feet.
Content the true way, 81,74 feet.

5th. OF ROUND TIMBER, WHOSE BASES ARE UNEQUAL.

The usual way to measure round timber I have shewn, and its error in timber that is all the way of an equal thickness; and it must be much more so in timber that is tapering; and the more tapering it is, the greater is the error. Therefore, to measure all such timber according to art and truth, such a piece ought to be considered as a frustum of a cone, and should be measured by the rule for the frustum of a cone.

10. If a piece of timber be 9 inches in diameter at the lesser end, and 36 inches at the greater end, and 24 feet long, how many feet of timber are therein?

Ans. 74,22 feet.

11. If a piece of timber be 136 inches in circumference at one end, 32 inches in circumference at the other end, and 21 feet long, how many feet of timber are contained in that piece? Ans. 92,34 feet.

6th. OF THE FIVE REGULAR BODIES.

These bodies may all be measured by the rule laid down for measuring a pyramid, except it be a cube, or hexaedron, which is already measured.

1st. OF A TETRAEDRON.

A tetraedron is a solid contained under four equal and equilateral triangles.

Required the solidity of a tetraedron, whose side is 12 inches, and the perpendicular height 9,798 inches.

Ans. 203,641632 inches.

2nd. OF AN OCTAEDRON.

The octaedron is a body contained under eight equal and equilateral triangles.

RULE. An octaedron is composed of two quadrangular pyramids, joined together by their bases; therefore, if the area of the base be multiplied into a third part of the length of both pyramids, the product will be the solid content.

Required the solid content of an octaedron, whose side is 12 inches? Ans. 814,5792 the solidity.

3d. OF A DODECAEDRON.

A dodecaedron is a solid body, contained under twelve pentangular planes, whose vertexes all meet in the centre.

RULE. Find the content of one of the pyramids, and multiply that by 12, (the number of pyramids in the dodecaedron,) and that product will be the solidity of the dodecaedron.

Let each side of a dodecaedron be 12 inches; required the solidity thereof? Ans. 13241,85392 inches.

4th. OF AN ICOSAEDRON.

The icosaedron is a solid body, contained under twenty equal and equilateral triangles. The icosaedron is composed of twenty triangular pyramids, with their vertexes all joined in the centre.

RULE. Find the solid content of one pyramid, multiply that by twenty, and the product is the solid content of the icosaedron.

Let each side of an icosaedron be 12 inches; required the solidity thereof? Ans. 3769,945840 inches.

NOTE. The cube, or hexaedron, is already measured.

By the following table, the contents, either superficial or solid, of any of these bodies may very readily be found; for all like superficial figures are in proportion one to another, as are the squares of their like sides: therefore it will be, as the square of 1, (which is 1,) is to the superficial content in the table, so is the square of the side of the like body to the superficial content of that body. Therefore, if the number in the table be multiplied by the square of the side given, the product is the superficial content required.

Again, all like solids are in such proportion to each other, as are the cubes of their like sides: therefore it will be, as 1, (which is the cube of 1,) is to the solid content in the table, so is the cube of the side given to the solid content required. Therefore, if the number in the table be multiplied by the cube of the given side, the product will be the solid content of the same body.

A Table shewing the solidity and superficial content of any of the regular bodies, the side being 1, or unity.

Names of bodies.	The solidity.	The superficies.
Tetraedron	0,1178511	1,732051
Octaedron	0,4714045	3,464102
Hexaedron	1,0000000	6,000000
Icosaedron	2,181695	8,660254
Dodecaedron	7,663119	20,645729

I will give two examples, according to the tabular numbers.

1. Let the side of a dodecaedron be 12 inches, (as before;) what is the content, solid and superficial?

Ans. 13241,869632 the solid content.
2972,984976 the superficial content.

2. Let the side of an octaedron be 20 inches; what is the content, solid and superficial?

Ans. 3771,2860000 the solid content.
1385,640800 the superficial content.

7th. HOW TO MEASURE ANY IRREGULAR SOLID.

If you have any piece of wood or stone that is craggy or uneven, and you desire to find the solidity of it, put the body into any regular vessel, as a tub, a cistern, or the like, and pour in as much water as will just cover it; then take out the solid, and measure how much the fall of the water is, and so find the solidity of that part of the vessel.

EXAMPLE.

Suppose a piece of wood or stone to be measured; and suppose a tub 32 inches in diameter, into which is set the stone or wood, and covered with water. When the solid is taken out, the fall of the water is 14 inches. I demand the solidity of that body? Ans. 6,51 feet.

PRACTICAL QUESTIONS.

1. If a pavement be 47 feet 9 inches long, and 18 feet 6 inches broad, I demand how many yards are contained therein? Ans. 98 yards 1 foot.

2. There is a room whose length is 21,5 feet, and the breadth 17,5 feet, which is to be paved with stones, each

1 foot 6 inches square. I demand how many such stones will pave it? Ans. 167 stones.

3. There is a room 109,75 feet about, and 9,25 feet high, which is all, (except two windows, each 6 feet 6 inches high, and 5 feet 9 inches broad,) to be hung with tapestry an ell broad. I want to know how many yards will hang said room? Ans. 83,59 yards.

4. If the axis of a globe be 27,5 inc. I demand the contents, solid and superficial?

Ans. 6,3 feet solid, and 16,49 feet superficial.

5. There is the frustum of a globe, the diameter of whose base is 24 inches, and the altitude thereof is 19 inches. I demand the contents, solid and superficial?

Ans. 2785,5520 inches solidity.
1218,9408 inches superficial.

6. If a tree girth 18 feet 6 inches, and 24 feet long, how many tons of timber are contained in that tree?

Ans. 12 tons 33 feet 4½ inches.

7. There is a cellar to be dug by the floor, whose length is 33 feet 7 inches, and the breadth 18 feet 9 inches: I demand how many floors of earth are in that cellar?

Ans. 11 floors 56 feet.

NOTE. 18 feet square, and 1 foot deep, is a floor of earth.

8. There is a roof to be covered with shingles, whose depth on both sides is 35 feet 6 inches, and the length 48 feet 9 inches. How many squares of shingling are contained therein? Ans. 17 squares 30 feet.

9. There is a cone, whose diameter at the base is 42 inc. and the perpendicular height 94 inches, and it is required to cut off two solid feet from the top-end thereof. I demand what length upon the perpendicular must it be cut off?

Ans. 40,43 inches.

10. If a square piece of timber be 12 feet long, and if the side of the square of the greater base be 21 inches, and the side of the square of the lesser base 3 inches, how far must I measure from the greater end, to cut off three solid feet?

Ans. 22,46 inches.

11. Three men bought a grindstone, of 40 inches diameter, which cost 20 dollars, of which sum the first man paid 9 dollars, the second 6, and the third 5. I demand how much of the stone each man must grind down, proportionally to their money?

Ans. 5,17 inches, breadth of ring for the first.
4,83 inches, breadth of ring for the second.
10,00 inches, breadth of ring for the third.

12. How many feet of square-edged boards, 1¼ inches thick, including the saw-calf, can be sawn out of a log 20 feet long, and 24 inches diameter? Ans. 384 feet.

RULE. As the thickness of the board, including the saw-calf, is to the solid content, so is 12 inches to the answer.

13. If the diameter of a round tree, (equally thick from end to end,) be 22 inches, and its length 20 feet, I demand how many solid feet it will contain, when hewn square?
Ans. 33,6+ feet.

RULE. Multiply twice the square of its semidiameter in inches, by the length in feet, and divide that product by 144, and the quotient will be the answer.

14. A gard'ner had an upright cone,
Of which to cut a rolling-stone,
The largest it would make:
The mason said there was a rule
For such work, but he'd a thick skull.
Help him for pity's sake.
Ans. It must be cut at ⅓ of its height.

15. There is a corn-crib, whose depth is seven-tenths of the width, and the length six times the depth, and the solid capacity is 367½ feet. I demand the depth, breadth, and length, and how many bushels of corn it will contain?
Ans. 3 ft. 6 inc. the depth, 5 ft. breadth, 21 ft. length.
It will contain 295+ bushels.

16. Suppose, sir, a bushel is exactly round,
Whose depth being measur'd 8 inches I found.
If the breadth 18½ inches you discover,
That bushel is legal all America over.
But a workman would make one of another frame;
Inches 7½ must be the depth of the same.
Now, sir, of what length must the diameter be,
That it may with the former in measure agree?
Ans. 19,107 inches diameter.

17. In the midst of a meadow, well cover'd with grass,
I took just an acre to tether my ass:
How long must the cord be, that, feeding all round,
He may'nt graze more or less than his acre of ground?
Ans. 117½ feet long.

18. A farmer has a corn-crib 16½ feet square, but he has a mind to pull it down, and build a new one large enough

to hold three times as much as the old one. I demand how much square the new one must be?

Ans. 28 feet, and near 7 inches.

19. There is a stone 20 inches long, 15 inches broad, and 8 inches thick, which weighs 217 lbs. I demand the length, breadth, and thickness of another of the same kind and shape, which weighs 1000 lbs.?

Ans. Length 33,28 inches, breadth 24,96 inches, and thickness 13,312 inches.

20. If an iron bullet, whose diameter is 4 inches, weighs 9 lbs. what will the weight be of another bullet, of the same metal, whose diameter is 9 inches?

Ans. 102 lbs. 8 oz. 4 drs.

21. There is a square pyramid of marble, each side of its base is 5 inches, and the height thereof 15 inches, and its weight is 12 lbs. 4 oz. I demand the weight of another like square pyramid, each side of whose base is 30 inches?

Ans. 2646 lbs.

22. There is a globe of marble, whose diameter is 6 inc. and its weight 11 lbs. What will be the diameter of another globe, of the same marble, that weighs 500 lbs.?

Ans. 21,4 inches diameter.

23. There is a frustum of a pyramid, whose bases are regular octagons: each side of the greater base is 21 inches, each side of the lesser base 9 inches, and its length 15 feet. I demand how many solid feet are contained therein?

Ans. 119,2 solid feet.

24. There is a frustum of a cone, and the diameter of the greater base is 36 inches, the diameter of the lesser base 20 inches, and the length or height 215 inches. I demand the length and solid content of the whole cone, and also the solid content of the given frustum?

Ans. Content of the whole, 94,98 feet.
Content of the top-piece, 16,28
Content of the frustum, 78,70

25. If the top-part of a cone contains 26171 solid inches, and 200 inches in length, and the lower frustum thereof 159610 solid inches, I demand the length of the whole cone, and the diameter of each base?

Ans. Length of the whole cone, 384,3 inches.
Diameter of the greater base, 42,94
Diameter of the lesser base, 22,35

26. There is a frustum of a cone, whose solid content is 20 feet, and its length 12 feet, and the greater diameter

bears such proportion to the lesser, as 5 to 2. I demand the diameters?

Ans. The greater diameter is 24,24147 inches, and the lesser 9,69659 inches.

27. There is a wall which contains 18225 cubic feet, and the height is five times the breadth, and the length eight times the height. I demand the length, height and breadth?

Ans. Length 180, height 22½, and breadth 4½ feet.

28. There was a pole, whose top-end was broken off by a blast of wind, and in falling struck the ground at 15 feet distance from the lower end of the pole: the broken piece was 39 feet long. I demand the length of the pole?

Ans. 75 feet.

By Euclid, book first, 47th proposition, the square of the hypothenuse of any right-angled triangle, is equal to the sum of the squares of the base and perpendicular.

29. A May-pole there was, whose height I would know:
The sun shining clear, straight to work I did go.
The length of the shadow upon level ground,
Just sixty-five feet, when measur'd, I found.
A staff I had there, just five feet in length,
The length of its shadow was four feet one-tenth.
How high was the May-pole I gladly would know,
And it is the thing you are desired to shew?

Ans. 79,26+ feet.

30. What will be the diameter of a globe, when the solidity and superficial content thereof are equal? Ans. 6.

31. What will be the axis of a globe, when the solidity is in proportion to the superficies, as 18 to 8? Ans. 2⅔.

Note. Nos. 30 and 31 are taken from Algebra.

32. Suppose a pole on level ground to be 75 feet above the surface, at what height from the ground must I cut it, that the top-end may fall 55 feet from the lower end of the pole, the end where it was cut off resting on the stump or upright part? Ans. 17½ feet high.

33. There are three Grenado shells of such capacity, that the second shell will just lie in the concavity of the first, and the third in the concavity of the second. The solidity of the metal of the first shell is equal to its concavity, and the solidity of the metal of the second to the concavity, is as 7 to 5, and the solidity of the third or least shell's metal to its concavity, is as 9 to 4. Now supposing the diameter of the first or greatest shell to be 16 inches, and allowing every solid inch of iron to weigh 4 oz. what is the diameter

of the two lesser shells, and the thickness and solidity of the metal of every shell, and the weight of every shell?

Ans. Weight of the greatest, 268,08 lbs. of the second, 156,38 lbs. and of the least, 77,38 lbs.
Diameter of the second shell, 12,699 inches, and of the least shell, 9,485 inches.
The thickness of the metal of the greatest shell, 1,65 inches, of the second shell, 1,607 inches, and of the third shell, 1,541 inches.

34. I have a joist 7,5 inches wide, and 2,25 inches thick, but I want one twice as large, that will be 3,75 inc. thick. I demand the breadth of such an one? Ans. 9 inc.

35. I have a square stick of timber 18 inches by 14, but one of a third part of the timber in it, provided it be 8 inc. deep, will serve: how wide will it be? Ans. 10,5 inc.

36. I have a plank 16 feet 5 inches long, and I want just a square yard slit off: at what distance from the edge must the line be drawn? Ans. $6\frac{11}{197}$ inches.

37. I have a square garden, whose area is 2500 yards: what is each side of the square, and the breadth of a walk along one side, one end of which may just take up one-half of the square?

Ans. One side of the square is 50 yards.
Breadth of the walk is 14,65 yards.

38. What difference is there between one floor 20 feet square, and two others, each 10 feet square?

Ans. 200 feet difference.

39. What is the difference between a solid half foot, and half a foot solid?

Ans. 648 inches: or the half a foot solid is only the quarter of a solid half foot.

40. A fellow said that when he counted his basket of peaches, 2 by 2, 3 by 3, 4 by 4, 5 by 5, 6 by 6, there was one left; but when he counted them 7 by 7, they came out even. How many peaches had he? Ans. 103.

41. A hare starts 12 rods before a greyhound, and is not perceived by him, till she is up 45 seconds: she scuds away at the rate of 10 miles an hour, and the dog in view makes after her, at the rate of 16 miles an hour. How long will the course hold, and what space will be run over, from the spot where the dog started?

Ans. 1 min. $37\frac{1}{2}$ sec. and the space ran by the dog was 3 furlongs $18\frac{3}{4}$ rods.

42. Suppose I send a man from here. (Washington, G. C.) at 6 o'clock in the morning, to Pittsburgh, distant 100 miles,

who is to travel at the rate of 4 miles an hour, without intermission. At 3 o'clock in the afternoon of the same day, a messenger leaves Pittsburgh for this town, who is to travel at the rate of 7 miles an hour. Whereabouts on the road will they meet?

Ans. The messenger from Washington will meet the messenger from Pittsburgh, $59\frac{3}{11}$ miles from Washington; and the messenger from Pittsburgh will meet the messenger from Washington, $40\frac{8}{11}$ miles from Pittsburgh.

43. A and B are on opposite sides of a circular field, 300 rods about: they begin to go round it at the same time, and the same way. A goes 22 rods in two minutes, B 34 rods in three minutes. How many times will they go round the field, before B overtakes A? Ans. 17 times round.

44. A water tub holds 100 gallons; the pipe which conveys the water to it, admits 7 gallons in 5 minutes, and the tap discharges 20 gallons in 17 minutes. Now supposing both of these to be left open intentionally, and the water to be turned on at 4 o'clock in the morning, a servant, at 6 o'clock, observes the water running, and puts in the tap. In what time after this accident will the tub be full?

Ans. $51\frac{11}{11}$ min. past 6 o'clock.

45. A gay young fellow soon got the better of $\frac{5}{7}$ of his fortune: he then gave 6000 dollars for a commission in the army, and continued his extravagancies till he had but 1800 dollars left, which he found to be but $\frac{3}{8}$ of his money, after he had purchased his commission. What was his fortune at first? Ans. 15120 dollars.

46. Suppose a bar of steel to weigh 12 lbs. in one scale, and in another 10 lbs. what is the true weight of the steel?

Ans. 10,954 lbs.

47. A merchant was possessed of $\frac{2}{3}$ of a ship, and sold $\frac{5}{8}$ of his interest for 1500 dollars. I demand the full amount of the ship, at that rate? Ans. 6000 dollars.

48. In an orchard of fruit-trees, $\frac{1}{4}$ of them bears pears, $\frac{1}{3}$ apples, $\frac{1}{6}$ plums, and 50 of them peaches, and 30 of them cherries. How many trees are in the orchard of all sorts?

Ans. 960 trees.

49. A, B and C, have among them 200 dollars. A's and B's is to B's and C's as 5 to 7; and C's, less B's, is to C's and B's as 1 to 7. How many dollars had each of them?

Ans. A had $\$44\frac{4}{9}$, B $\$66\frac{6}{9}$, and C $\$88\frac{8}{9}$.

50. In a series of proportional numbers, the first is 4, the third is 12, and the product of the second and third is 112,8. I demand the difference of the second and fourth?

Ans. 18,8 difference.

51. There is an island 40 miles in circumference, and three men start together to travel round it: A goes 7 miles a day, B 8, and C 9. When will they all come together?

Ans. In 40 days; and A will travel 280 miles, B 320 miles, and C 360 miles.

52. What number is that, from which, if you take $\frac{3}{4}$, the remainder will be $\frac{1}{3}$? Ans. $\frac{22}{33}$.

53. A gentleman making his addresses in a lady's family, who had five daughters, she told him that their father had made his will, which imported, that the first four of the girls' fortune together was 20000 dollars, the last four was 26400 dollars, and the three last, with the first, was 24000 dollars; and the three first, with the last, made 22400 dollars, and the two first, with the two last, was 25600 dollars, which, if he could unravel, and make appear what each is to have, as he had a partiality for Charlotte, the second daughter, he should be welcome to her. What was her fortune, and that of each separately?

Ans. First daughter's fortune was $3200, Charlotte's was $5600, third daughter's, $4000, fourth daughter's, $7200, fifth daughter's, $9600.

54. Being 16 miles 4 furlongs from Baltimore, I saw with my telescope, in the dusk of the evening, the lamp of the lighthouse in the horizon, at which time my eye was six feet above the surface of the water. What is the height of the lighthouse above the water? Ans. 43 yards nearly.

55. A, B and C, found a purse of money, containing 234 dollars, and agreed to have it so divided, that when A's is multiplied by 2, B's by 3, and C's by 4, their products will be alike. I demand each man's share of the money?

Ans. A's share $108, B's $72, and C's $54.

56. A gambler loses, in four turns of dice, 320 dollars, and tripled each turn the sum he put in. I demand how much he played for the first and last time?

Ans. $8 the first time, and $216 the last time.

57. If a bear can eat a goose in 10 minutes, a wolf in 20 minutes, and a fox in 40 minutes, how long would they all be in eating it, if let at it together? Ans. 5 min. $42\frac{6}{7}$ sec.

58. Divide 20 dollars between four persons, giving one the third, another the fourth, another the fifth, and the fourth man one-sixth. What is each man's share?

Ans. The first got $7\frac{1}{7}$, the second $5\frac{1}{7}$, the third $4\frac{3}{7}$, and the fourth $3\frac{2}{7}$ dollars.

59. When half the number of an assembly, and 20, were met, there were a third and 10 absent. How many did the branch of the legislature consist of? Ans. 180 members.

60. If three unequal vents will severally empty a vessel of 120 gallons in one hour, two hours, and three hours, in how long would they empty it, if all running at once?
Ans. 32 min. $43\frac{7}{11}$ sec.

61. What weight of lead will cover a house, the area of whose roof is 6000 feet, and the thickness of the lead $\frac{1}{144}$ part of a foot? Ans. 15 tons 15 cwt. 3 qrs. 26 lbs. 10 oz.

62. Suppose the length of a sluice to be 30 feet, and the depth of water 4 ft. 6 inc. I demand the pressure against the side of the sluice? (See page 207.)
Ans. 7 tons 10 cwt. 2 qrs. 19 lbs.

63. Suppose a vessel 3 feet wide, 5 feet long, and 4 feet deep, I demand the pressure on the bottom of that vessel, it being filled with water to the brim? (See page 207.)
Ans. 1 ton 13 cwt. 1 qr. 26 lbs.

64. If a stream of clear water moves at the rate of 100 yards per minute, and meets with a fixed obstacle, 20 feet wide, and 3 feet high, I demand the momentary pressure of the stream? (See page 207.) Ans. 7312½ lbs.

NOTE. Water loses two-thirds of its power, in producing effects, being a yielding substance.

65. Let the squares of two numbers be 3161, (that is, their sum,) and their half sum, 1560¼. I demand those numbers? Ans. Greater 44, lesser 35.

66. Suppose the velocity of a cannon-ball to be 600 feet per second, I demand from what height must a body fall, to acquire the same velocity? Ans. 5625 feet.

67. If the diameter of a round stick of timber be 24 inc. from end to end, and its length 20 feet, I want to know how many solid feet it will contain, after it is hewn square, and what will be the contents of the slabs, which reduced it to a square? Ans. 40 feet the contents, when squared.
22,8 feet the contents of the slabs.

68. If a round stick of timber, 9 inches in diameter, contains 5 solid feet, I demand the diameter of one such stick, of the same length, which will contain ten times as much?
Ans. 28,46 inches.

69. Suppose I am getting a chaise made, and direct that the length of the shafts, between the axle-tree and backband, be 8 feet, and a dispute arises, whereabouts on the shafts the centre of the body should be fixed. The chaise-maker advised to place it 30 inches before the axle-tree, but others supposed 20 inches would be a sufficient encumbrance for the horse. Now supposing two passengers to weigh 336 lbs.

and the body of the chaise 84 lbs. more, I demand what will the horse bear in both cases more than his harness?

Ans. By the chaise-maker's directions, 131¼ lbs.
By the by-standers' directions, 87½ lbs.

70. A, B and C, can do a piece of building in 15 days: A can finish or do it in 30 days, and B in 40 days. I demand in what time can C finish it alone? Ans. 120 days.

71. A and B can finish a piece of work in 20 days, and with C they can do it in 12. In what time can C do it by himself? Ans. 30 days.

72. If tea is sold at 1 dol. 20 cts. which cost 1 dol. 25 cts. a pound, what is the loss per cent.? Ans. 4 per cent.

73. If by selling a yard of cloth for 2 dollars 30 cents, I gain 15 per cent. what would I gain per cent. by selling the same cloth at 2 dollars 40 cents per yard?

Ans. 20 per cent.

74. There are two numbers, whose product is 1610; the greater is 46. What is the sum of their squares, and what the cube of their difference?

Ans. 3341 the sum of their squares.
1331 the cube of their difference.

75. There is a mast or pole, $\frac{1}{5}$ of whose length stands in the ground, 12 feet of it in the water, and $\frac{2}{3}$ of its length in the air, or over water. I demand the whole length?

Ans. 216 feet.

76. There are two pieces of linen, one of which is 9 yards shorter than the other, and cost 12 dollars 24 cents: the other piece at the same price cost 14 dollars 40 cents. I demand how many yards are in both pieces, and the price of one yard? Ans. 111 yds. in both pieces.
24 cents per yard.

77. In 81034 kilderkins of wine, each 18 gallons, how many gross of bottles, each $\frac{3}{5}$ of a quart?

Ans. 45581 gross 7½ dozen.

78. Col. Beatty, of Cambridge, Ohio, being desirous to know how much money the toll of Cambridge bridge would amount to in a certain time, gave orders to David Ballantine, to receive of every wagon 25 cents, of every horseman 12½ cents, and of every footman 6¼ cents. Now, at the end of the year, Ballantine brings his account to Col. Beatty, amounting to 1325 dollars, and tells him, that as often as 5 passed with wagons, 9 passed on horseback, and as often as 6 passed on horseback, 10 passed on foot. I demand how many passed with wagons, how many on horseback, and on foot? Ans. 2000 wagons, 3600 horsemen, 6000 footmen.

79. Five merchants, A, B, C, D and E, have gained 2000 dollars, which they divide in such a way, that $\frac{1}{2}$ the share of A is equal severally to $\frac{1}{3}$ of B's, $\frac{1}{4}$ of C's, $\frac{1}{5}$ of D's, and $\frac{1}{6}$ of E's. What was each merchant's share?

Ans. A's $160, B's 320, C's 400, D's 480, E's 640.

80. Twenty admirals, 30 post-captains, 24 sailing-masters, and 24 lieutenants, spent 256 dollars at a ball, which sum was divided in such a manner, that 4 admirals paid as much as 5 post-captains, and 10 post-captains as much as 16 sailing-masters, and 8 sailing-masters as much as 12 lieutenants. I want to know the sum of money paid by the admirals, post-captains, sailing-masters, and lieutenants?

Ans. The 20 admirals paid $80, the 30 post-captains $96, the 24 sailing-masters $48, and the 24 lieutenants $32.

81. A, B and C, will make 20 rods of the turnpike road in 12 days; B, C and D, in 14 days; C, D and A, in 15 days; and D, A and B, in 18 days. In what time will it be done by all of them working together, and by each singly?

Ans. By A in 47,848 days, by B in 38,960 days, by C in 27,194 days, by D in 111,176 days, and by A, B, C and D, in 10,8309505 days.

82. Suppose A and B to have between them a certain number of dollars, to be so divided, that the sum of their squares may be 208, and the difference of their squares 80. Now supposing A to have the greater quantity, how many had he more than B?

Ans. A had 4 more than B; A's number 12, B's 8.

83. B and C had 50 dollars between them, which was to be so divided, that the sum of their squares should be 1300 dollars. How many had each, supposing B to have the greater quantity? Ans. B had 30, and C 20 dollars.

84. Divide 316 dollars 96$\frac{3}{4}$ cents between 4 men, 6 women, and 9 boys, giving each man as much as two women, and each woman as much as two boys. What is each of their shares?

Ans. Each man got 34 dollars 26$\frac{3}{4}$ cents, each woman 17 dolls. 13$\frac{1}{2}$ cts. and each boy 8 dolls. 56$\frac{3}{4}$ cts.

85. What two numbers being added will make 266$\frac{2}{3}$, and when one is multiplied by 3, and the other by 5, their products will be equal? Ans. 166$\frac{2}{3}$ and 100.

86. A man married a wife, and when the wedding day came, he was just double her age. After they had lived together 30 years, his age was in proportion to her's, as

2 to $1\frac{3}{7}$; and 30 years after, his age was to her's, as 2 to $1\frac{3}{5}$: then they both died. I demand their different ages?

Ans. His age 40 years, and her's 20, when married.

When they had lived together 30 years, his age was 70 yrs. and her's 50, making his age to her's, as the proportion of 2 to $1\frac{3}{7}$: and 30 years after that, his age was 100 years, and her's 80, making a proportion of his to her's, as 2 to $1\frac{3}{5}$.

87. The hour and minute hands of a watch or clock, are together precisely at 12 o'clock: when are they next together?

Ans. At 5 min. $27\frac{3}{11}$ sec. past 1 o'clock, at 10 minutes $54\frac{6}{11}$ sec. past 2 o'clock, at 16 min. $21\frac{9}{11}$ sec. past 3 o'clock, &c.

88. A number of dollars are to be divided between two men, A and B, in such a manner, that A may have 50 more than B, and that the sum of the squares of their respective shares may be 12500 dolls. I demand each man's number?

Ans. A had $100, and B 50.

89. Tom and Jack, playing at cards, having at first 14 dollars apiece, but after playing several games, Jack, having lost some of his, would play no longer; and it was found that the difference of the squares of the numbers that each then had, was 336. Pray how much did Jack lose?

Ans. 6 dollars.

90. Said William to James, my father gave me 12 cents more than he gave my brother John, and the difference of the squares of our separate parcels is 288. Now tell how many he gave us each?

Ans. William 18 cents, and John 6 cents.

91. On how much ground do 2016 soldiers stand, by placing 56 in rank, and 36 in file, and to stand 4 feet distance in rank, and the same in file? Ans. 30800 square feet.

92. I want to plant an orchard of 600 apple-trees, so that the length shall be to the breadth, as 3 to 2; and the distance of each tree, one from another, may be 7 yards. How many trees must be in length, how many in breadth, and how many square yards of ground do they stand on?

Ans. 30 in length, and 20 in breadth, and standing on 26999 square yards.

93. If a pipe, 6 inches bore, will discharge a certain quantity of water in 4 hours, in what time will 3 pipes, each 4 inches, discharge double as much water? Ans. 6 hours.

94. Suppose a ladder 60 feet long be so placed, as to reach a window 37 feet from the ground, on one side of the street, and by moving or turning the ladder, (not at the foot,) over,

will reach a window 28 feet high, on the other side of the street. What is the breadth of the street?

Ans. 102,64 feet.

95. Suppose a boat should set out from Wheeling, to go down the river 20 miles, and another boat start 20 miles below Wheeling, to come up the river to Wheeling, at the same moment of time; and suppose the current forwards one, and retards the other a mile and a half an hour. The boats are equally laden, and the rowers equally good, and in the common way of rowing in calm water, each would proceed at the rate of four miles an hour. How far from Wheeling will the two boats meet?

Ans. The boat from Wheeling will meet the boat going to Wheeling 18¾ miles below Wheeling, and the boat going to Wheeling will then be 6¼ miles from where she started.

96. I want to know how long a blacksmith would make a rod, or bar of iron, fourth of an inch square, out of a solid foot of iron, allowing no waste for the fire?

Ans. 576 feet long.

97. A country spark paid his addresses to a young lady versed in numbers, and asked her age, to which she replied, that if her age was multiplied by 3, and $\frac{3}{4}$ of that product tripled, the square root of $\frac{4}{9}$ of that product will be 4. Now kind sir how old am I? Ans. 14 years.

98. What are those two numbers, whose sum is 48, and their difference 14? Ans. 31 and 17.

99. Suppose a tavern signpost to be 40 feet above the surface of the ground, and it was observed to incline to fall, occasioned by the wind blowing hard, one foot from the top-end in 24 hours, and the next 24 hours one foot more, and so on, every 24 hours one foot, until such time as it lay flat on the level ground. How long was the signpost in falling, at that rate? Ans. 62 days 20 hours 34¾ minutes.

100. William Rabe, merchant of this town, (Washington, Guernsey county, Ohio,) has engaged three horses, belonging to different men, A, B and C, to haul a load of tobacco to Wheeling, for ten dollars. A and B are supposed to haul $\frac{3}{11}$ of the load, and A and C $\frac{5}{13}$. B and C can haul $\frac{3}{7}$ of the load. They are to be paid according to their work. How much is each to get of the ten dollars?

Ans. A is to get 4 dolls. $21\frac{2955}{4715}$ cts. B 3 dolls. $94\frac{2290}{4715}$ cents, and C 1 dollar $84\frac{1440}{4715}$ cents.

101. The same merchant, William Rabe, bought three horses from different men for 200 dollars; one of them from Jacob Endley, one from Samuel Laurence, and one from

William Skinner. They were to be paid for according to their several abilities in hauling, and in order to determine their prices, they were proved in the following manner: Jacob Endley's horse hauled a certain load of salt, from Cambridge to Washington, in ten hours. Jacob Endley's horse, and Samuel Laurence's horse, will haul the same quantity of salt from the same place in six hours. Jacob Endley's horse, and William Skinner's horse, will do the same work in four hours. Now I demand who owned the best horse, and the several prices paid to each owner?

Ans. Wm. Skinner received 94 dolls. 73 cts. $6\frac{16}{19}$ m.
Jacob Endley received 63 dolls. 15 cts. $7\frac{17}{19}$ m.
Samuel Laurence rec'd. 42 dolls. 10 cts. $5\frac{5}{19}$ m.

102. Suppose a body to weigh 400 lbs. at 2000 miles distance above the surface of the earth, what would the same body weigh on or at the surface, estimating the earth's semi-diameter at 4000 miles? Ans. 900 lbs.

103. If a body on the surface of the earth weighs 180 lbs. how high must it be raised in the air, to weigh only 20 lbs.?

Ans. 8000 miles high.

104. How high will a rifle-ball, that runs 120 to a pound, go in the air, to lose half its weight?

Ans. 1656,85 miles high.

105. A can finish a certain length of the national turnpike road in 13 days; A and B together in 8 days. In what time can B do it alone? Ans. $20\frac{4}{5}$ days.

106. A tract of land is to be laid out in form of an equal square, and to be enclosed by a fence of post and rails, of five high, so that each rod of fence shall contain ten rails. How large must the square be, to contain just as many acres of land as there are rails in the fence that encloses it, so that every rail shall fence an acre?

Ans. 256000 rails, which encloses 256000 acres of land, and the ground or tract will be 20 miles square.

107. If 6 lbs. of pepper be worth 13 lbs. of ginger, and 19 lbs. of ginger be worth 4 lbs. 12 oz. of cloves, and 10 lbs. of cloves be worth 63 lbs. of sugar, at 5 cents a pound, what is the value of 100 lbs. of pepper?

Ans. 17 dollars $6\frac{1}{4}$ cents.

108. If when wine is 70 dollars per hogshead, a company of 45 persons will spend 80 dolls. therein, in a certain time, what is the wine a pipe, when 13 persons more will spend 252 dollars in twice the time, drinking with equal moderation? Ans. 171 dollars 7 cents nearly.

109. William Rabe despatches his clerk from Washington to Philadelphia, distant 400 miles, with orders to purchase goods, his route fixed at 22 miles a day. Four days after, he sent a messenger after him with fresh orders, and is to travel at the rate of 32 miles a day. How many miles this side of Philadelphia will the messenger overtake the clerk?

Ans. $118\frac{2}{3}$ miles.

110. A merchant sold cloth at 2 dollars 20 cents a yard, by which he cleared $\frac{3}{8}$ of his money; but growing scarce, he raised the same cloth to 2 dollars 70 cents per yard. I demand his profit per cent. at the latter price?

Ans. 96 dollars $36\frac{4}{11}$ cents per cent.

111. A man dying left his wife with child, and by his will ordered, that if she had a son, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother: and if she had a daughter, the mother was to have $\frac{2}{3}$, and the daughter $\frac{1}{3}$. But it happened she was delivered of both a son and daughter, by which she lost in equity 8000 dollars more than if it had been only a girl. I demand how much would have been her dowry, if she had only a son? Ans. 7000 dolls.

112. A, B and C, put in a common stock 15440 dollars, to trade in company. A's was in 3 months, B's 5 months, and C's 7 months, and they gained 986 dollars, which was to be so divided, that $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's, and $\frac{1}{3}$ of B's to $\frac{1}{4}$ of C's. I demand how much each put in stock, and each man's gain?

Ans. A put in $5600, B 5040, and C 4800. A gained $208, B gained $312, and C gained 416.

113. A man married a wife, and on his wedding day, his age was to his wife's age, as 3 to 1; and when they had lived together 15 years, his age then was to his wife's age, as 2 to 1. I demand both their ages when married?

Ans. He was 45, and his wife 15.

114. A father divided a sum of money between his three sons, A, B and C; he gave A 7 dollars as often as B 4; to C he gave 2 as often as B 5; and the dividend of C came to 2000 dollars. What was each son's share, (that is, A's and B's,) and the whole legacy?

Ans. A's share was $8750, B's 5000, C's 2000, and the whole legacy was $15750.

115. Suppose in a room, where two men are sitting, A and B, there is a fire, from which A is 3 feet, and B 6 feet distant: it is required to find how much hotter it is at A's eat, than at B's? Ans. A is 4 times as warm as B.

116. Suppose the battering ram, used by the Turks, to weigh 8000 lbs. that it was moved 24 feet in a second, and that was sufficient to demolish the walls of ——, with what velocity must a cannon-ball be moved, which weighs 42 lbs. to do the same execution?

Ans. 4571$\frac{1}{7}$ feet in a second.

117. What quantity of water must be added to a pipe of wine, which cost 182 dollars, to reduce the first cost to 90 cents per gallon? Ans. 20$\frac{2}{3}$ gallons of water.

118. A man caught a fish, whose head was 8 inches long, and the tail as long as the head and half the body, and the body was just as long as the head and tail. What was the length of the fish? Ans. 64 inches.

119. Two merchants enter into partnership, each of them putting in 8 hhds. of tobacco, those of A costing 192 dollars more than B's. The tobacco being sold, they find they have gained 1000 dollars, of which B had, for his share, 450 dollars. The question is to know, at how much was each parcel of tobacco valued?

Ans. A's valued at $132, and B's at $108 per hhd.

120. If rum is sold at 50 dollars a hogshead, and there is lost 6 per cent. how much is the gain or loss, when 4 hhds. are sold for 180 dollars?

Ans. 15 dollars 59 cents per cent. loss.

121. What sum of money, at 3½ per cent. per annum, will clear 154 dolls. in 1 year and 8 months? Ans. $3520.

122. Having a desire to collect a number of men for a frolick, and wishing to keep them sober, I had a barrel containing 32 gallons of whiskey, out of which I took 5 gallons, and replaced it by putting in 5 gallons of water; and finding it too strong, took out 5 gallons more, putting in 5 gallons of water, and did so five times altogether, still minding to replace the 5 gallons taken out, by 5 gallons of water. I demand the quantity of whiskey and water in the barrel?

Ans. 18$\frac{717419}{1048576}$ gallons of whiskey, and 18$\frac{331157}{1048576}$ gallons of water.

123. What number is that, which being divided into four or five equal parts, the product of all the parts, in both cases, shall be exactly the same? Ans. 12$\frac{53}{256}$.

124. Bought a certain quantity of cloth for 2400 dollars; and if the number of dollars it cost per yard was added to the number of yards bought, the sum would be 604. I demand the number of yards bought, and at what price per yard? Ans. 600 yards, at 4 dollars per yard.

[The following I put in the Guernsey Times, a paper printed in Cambridge, Guernsey county, Ohio, July 2nd, 1825.]

A REBUS, BY JAMES L. CONNOLLY.

Two-fifths of a liquid that in Egypt is found,
One-fourth of a fruit that grows not far from the ground,
One-third of an article that's useful to man,
Two-thirds of a fluid that's us'd on the ocean,
One-fifth of a fruit that in Asia doth grow,
One-sixth of a river that in Europe doth flow,
One-seventh of a bird that is rare to be found,
One-third of a fruit that grows on American ground:
These letters, taken compactly, and put in a row,
Will spell a celebrated and virtuous hero.

Ans. WASHINGTON.

WINTER EVENING QUESTIONS.

1. Says John to Richard, I can place four 1's in such a manner, that they will make 3, 7, 8, 12, and even 20.

2. Three jealous husbands and their wives being ready to pass by night over a river, find at the waterside a boat which can carry but two persons at a time, and for want of a waterman, are obliged to row themselves over at several times. The question is, how these six persons shall pass by two and two, so that none of the three wives may be found in the company of one or two men, unless her husband be present?

3. A countryman in his journey, having a fox, a goose, and a peck of corn, came to a river, where it happened he could carry but one over at a time. Now as no two were to be left together, that would destroy each other, he was at his wits' end how to dispose of them; 'for,' says he, 'though the corn can't eat the goose, nor the goose the fox, yet the goose can eat the corn, and the fox the goose.' The question is, how must he carry them over, that they may not devour each other?

4. Suppose the nine digits to be placed in a quadrangular form, in what order must they stand, that any three figures in a right line may make just 15?

5. Two merry companions are to have equal shares of eight gallons of whiskey, which are in a vessel containing exactly eight gallons; and to divide it equally, they have only two other vessels empty, one of which contains five gallons, and the other three. The question is, how will they divide the said eight gallons of whiskey between them, by the help of these three vessels only, so that they may have four gallons each?

PART II.

APPENDIX.

OF PENDULUMS.

To find what length pendulums will be, to swing any given time.

RULE. Multiply the square of the seconds, in any given time, by 39,2, and the product will be the length in inches.

EXAMPLES.

1. Required the length of a pendulum that will swing quarter seconds? ,25×,25×39,2=2,45 inc. Ans.

2. Required the length of a pendulum that will swing half seconds? ,5×,5×39,2=9,8 inc. Ans.

3. Required the length of a pendulum that will swing seconds? 1×1×39,2=39,2 inc. Ans.

4. Required the length of a pendulum that will swing minutes?

60×60×39,2=141120 inc. or 2 miles 1200 feet. Ans.

5. Required the length of a pendulum that will swing hours?

3600 sec. equal 1 hour; then 3600×3600×39,2 equal the inches in 8018 miles, and 1152 inches over.

6. What is the difference between the length of a pendulum that swings half seconds, and one that swings three seconds? 3×3×39,2—,5×,5×39,2=28$\frac{7}{12}$ feet. Ans.

7. How often will a pendulum of 9$\frac{4}{9}$ inches vibrate in a second?

RULE. Divide the given length by 39,2, and the quotient will be the square of the time in seconds.

9$\frac{4}{9}$=9,8÷39,2=,25 or $\frac{1}{4}$ of a second. Ans.

OF THE LEVER.

To find what weight may be raised or balanced by any given power.

RULE. As the distance between the body to be raised or balanced, and the prop, is to the distance between the prop and the point where the power is applied, so is the power to the weight which it will raise or balance.

1. Suppose a man weighs 200 lbs. standing on the end of a lever 12 feet long, what weight will he balance on the other end, supposing the prop 1,5 feet from the weight?

12 feet equal the length of the lever.
1½ feet equal the distance of the weight from the prop.

10½ feet equal the distance from the prop to the man.

As 1½ : 200 :: 10½ : 1400 lbs. answer.

2. Suppose a body weighing 50 lbs. rests on the end of a lever 8 feet long, what weight will that body raise or balance on the other end, supposing the prop 2,5 feet from the weight to be raised?

8—2,5=5,5 Then as 2,5 : 50 :: 5,5 to 110 lbs. Ans.

OF THE WHEEL AND AXLE-TREE.

RULE. As the diameter of the axle-tree is to the diameter of the wheel, so is the power applied to the wheel to the weight suspended by the axle-tree.

1. I would have a windlass made in such a manner, that a pound applied to the wheel should be equal to 12 lbs. suspended from the axle-tree. Now supposing the diameter of the axle-tree 6 inches, required the diameter of the wheel?

Say inversely 12 : 6 :: 1 to 72 inc. diameter of the wheel. Ans.

2. Suppose the diameter of the wheel 72 inches, required the diameter of the axle-tree, so that a pound applied to the wheel may balance 12 lbs. on the axle-tree?

Say inversely 1 : 72 :: 12 to 6 inc. diameter of axle-tree. Ans.

3. Suppose the diameter of the axle-tree to be 6 inches, and that of the wheel 72 inches, what power applied at the wheel will balance 12 lbs. at the axle-tree?

Say inversely as 6 : 12 :: 72 to 1 lb. Ans.

4. Suppose the diameter of the wheel 72 inches, and that of the axle-tree 6 inches, what weight at the axle-tree will balance one pound at the wheel?

Say inversely as 72 : 1 :: 6 to 12 lbs. Ans.

To find the quantity of pressure against the sluice or bank which pens water.

RULE for question 62, page 196. Multiply the area of the sluice under water, by the depth of the centre of gravity, (which is equal to half the depth of the water,) in feet, and that product again by 62½, the number of pounds Avoirdupois in a cubic foot of fresh water, and the product will be the number of pounds required.

RULE for question 63, page 196. The perpendicular pressure of fluids on the bottom of vessels, is estimated by the area of the bottom, multiplied by the altitude of the fluid.

RULE for question 64, page 196. Find the fall by the next question, which will produce the given velocity; multiply that height by 62,5 lbs. Avoirdupois, for clean river water, by 63 lbs. for dirty water, and by 64 lbs. for sea water.

EXAMPLES.

1. The mean velocity of a stream is 12 feet per second, what is the perpendicular fall of the stream?

$12\times12=144\div64=2\frac{1}{4}$ feet, answer.

2. What is the velocity of water issuing from a head of 6 feet deep?

$6\times64=384$, square root whereof is 19,5+ feet. Ans.

3. If the velocity of a stream, issuing through the bulkhead of a mill, be 18 feet per second, what head of water is there? $18\times18=324\div64=5\frac{1}{16}$ feet. Ans.

4. A miller has a head of water 6 feet 3 inches above the sluice, how high must the water be raised above the opening, so that half as much again water may be discharged from the sluice in the same time?

RULE. The quantity of water discharged from a hole in a vessel, is as the square root of the height of water above the opening.

$\sqrt{6,25}=2,5$ and half as much again is $2,5+1,25=3,75$ for the square root of the required depth: then $3,75\times3,75=14,0625$ feet high. Ans.

5. I observed, while a stone was falling from a precipice, a string with a bullet at the end, which measured 25 inches to the middle of the ball, make five vibrations. What was the height from whence the stone fell?

$25\div39,2=,6377$ and $\sqrt{,6377}=,7985$ of a second, the time of one vibration, and $,7985\times5=4$ seconds nearly, the time of the stone's descent: then $4\times4=16$, and $16\times16=256$ feet. Ans.

RULE. Take a line of any length, and by the last question find the time from the dropping of the stone, till you hear it strike the bottom. Multiply 73088, (equal to 16×4×1142 feet, the distance sound moves in a second,) by the number of seconds, till you hear the stone strike the bottom, and to this product add 1304164, (equal to the square of 1142,) and from the square root of that sum take 1142, and divide the square of the remainder by 64, (that is, 16×4,) and the quotient will be the depth of the well in feet. Divide the depth by 1142, and the quotient will be the time of the sound's ascent, which being taken from the whole time, will leave the time of the stone's descent in seconds.

EXAMPLES.

1. Suppose I drop a stone into a well, and a line with a plummet, which measured to the middle of the ball 25 inc. made five vibrations, before I heard the stone strike the bottom of the well, I demand the depth, time of the stone's descent, and of the sound's ascent?

25÷39,2=,6377 and $\sqrt{}$,6377=,7985 of a second, and ,7985×5=4 sec. nearly, to the hearing of the stone strike: then $\sqrt{}$73088×4+1304164—1142=121,53 and 121,53×121,53÷64=230,77 feet, the depth, and 230,77÷1142=,2 of a second, the time of the sound's ascent, and 4—,2=3,8 seconds, the time of the stone's descent.

Ans. Depth 230,77 feet, time of the stone's descent 3,8 seconds, and the sound's ascent ,2 of a second.

2. A bullet dropped from the top of a building was found to reach the ground in 1½ seconds: I demand the height of the building?

RULE. The square root of the feet in the space fallen through, will ever be equal to 4 times the number of seconds the body has been falling: therefore multiply the time by 4, and the square of the product will be the space fallen through in the given time.

1½ sec.=1,5×4=6, and 6×6=36 feet high. Ans.

3. How many feet will a body fall in 6 seconds?

6×4=24, and 24×24=576 feet. Ans.

4. If a ball, discharged from a gun, returns to the earth in 14 seconds, how high did it ascend?

RULE. Ascending bodies are retarded in the same ratio that descending bodies are accelerated.

The ball being half of its time in its ascent, therefore, 14÷2=7, and 7×4=28; and 28×28=784 feet. Ans.

5. In how many seconds will a weight fall through a space of 10000 feet?

RULE. Four times the number of seconds in which the body was in falling, will ever be equal to the square root of the space in feet, through which it has fallen. Therefore divide the square root of space fallen through by 4, and the quotient will be the space in which it has fallen.

$\sqrt{10000}=100$, and $100\div 4=25$ sec. Ans.

6. How long must a bullet be in falling, to acquire a velocity of 192 feet per second?

RULE. Four times the number of seconds in which a body has been falling, is equal to an eighth of the velocity in feet per second, acquired at the end of the fall. Therefore divide the given velocity by 8, and a fourth part of the quotient will be the answer.

$192\div 8=24$, and $24\div 4=6$ seconds. Ans.

7. In what time will a musket-ball, dropped from the top of a steeple 625 feet high, come to the ground?

$\sqrt{625}=25$, and $25\div 4=6\frac{1}{4}$ seconds. Ans.

8. With what velocity will an iron-ball begin to descend per second, if raised 2000 miles above the surface of the earth?

RULE. As the square of the earth's semidiameter is to 16 feet, so is the square of any other distance, counting from the earth's centre inversely, to the velocity with which it begins to descend per second.

4000 miles equal the earth's semidiameter, and $4000+2000$ equal the distance from the earth's centre: then say inversely,

As $4000\times 4000 : 16 :: 6000\times 6000$ to $7\frac{1}{9}$ feet per second. Ans.

9. How high must a ball be raised above the surface of the earth, to begin to descend with a velocity of $7\frac{1}{9}$ feet per second?

As $16 : 4000\times 4000 :: 7\frac{1}{9}$ to 36000000 miles: then $\sqrt{36000000}=6000$ miles: then $6000-4000$ (the earth's semidiameter) $=2000$ miles above the surface of the earth, which is the answer.

10. If a musket-ball dropped from the top of a steeple 625 feet high in 6,25 seconds, required its mean velocity?

RULE. Divide the space fallen through by the number of seconds it was in falling, and the quotient will be the mean velocity.

$625\div 6{,}25=100$ feet per second. Ans.

11. If a ball falls through a space of 625 feet high in 6¼ seconds, with what velocity will it strike the ground?

RULE. The velocity acquired at the end of any period, is equal to twice the mean velocity with which it passed during that period: or multiply the space fallen through by 64, the square root of which product is the velocity sought.

$625 \div 6{,}25 = 100$; then $100 \times 2 = 200$ feet. Ans.

12. If a ball strikes the ground with a velocity of 50 feet per second, from what height did it fall?

RULE. Divide the square of the velocity by 64, and the quotient will be the height required.

$50 \times 50 = 2500$; then $2500 \div 64 = 39\frac{1}{16}$ feet. Ans.

OF WATER.

13. There is a sluice, one end of which is 3 feet lower than the other: required the velocity of the stream per second?

$3 \times 64 = 192$, and $\sqrt{192} = 13{,}85+$ feet. Ans.

The weight given to be raised by a balloon, to find its diameter.

RULE. As the specific difference between common and inflammable air is to one cubic foot, so is any weight to be raised to the cubic feet contained in the balloon. Divide the cubic feet by ,5236, and the cube root of the quotient will be the diameter required to balance it with common air; but to raise it, the diameter must be something greater.

EXAMPLE.

I wish to construct a spherical balloon of sufficient capacity to ascend with three persons, weighing 5 cwt. 2 qrs. 24 lbs. and the balloon and a bag of sand to weigh 2 qrs. 4 lbs. Required the diameter of the balloon?

By the table of specific gravities, a cubic foot of common air weighs 1¼ oz. Avoirdupois, and a cubic foot of inflammable air weighs three twenty-fifths of an ounce Avoirdupois: then 1¼ equals 1,25, and three twenty-fifths equals 0,12: then 1,25 less 0,12 equals 1,13, the difference between a cubic foot of common and inflammable air. 5 cwt. 2 qrs. 24 lbs. equal 640 lbs. and 2 qrs. 4 lbs. equal 60 lbs. Then $640 + 60 = 700$ lbs. and $700 \times 16 = 11200$ ounces.

Then as 1,13 : 1 : : 11200 to 9911,5044 cubic feet: then $\sqrt[3]{9911{,}5044} \div ,5236 = 26\frac{1}{2}\frac{3}{0}$ feet diameter. Ans.

The diameter of a balloon given to find what weight it is capable of raising.

RULE. Multiply the cube of the diameter by ,5236, and the product will be the content in cubic feet. Then say,

as one cubic foot is to the specific difference between common and inflammable air, so are the contents of the balloon to the weight it will raise.

EXAMPLE.

The diameter of a balloon is $26\frac{1}{2}\frac{3}{0}$ feet, what weight is it capable of raising?

$26\frac{1}{2}\frac{3}{0}$ = 26,65: then 26,65×26,65×26,65×,5236=9911,5044 cubic feet: then 1,25—0,12=1,13, the difference. Then say,

As 1 : 1,13 : : 9911,5044 to 11199,882 oz. equal to 700 lbs. nearly. Ans.

A Table of specific gravities, of several solid and fluid bodies: the second column contains their absolute weight, and the third their relative weight, in Avoirdupois ounces.

A cubic foot.	Abso.	Relat.	A cubic foot.	Abso.	Relat.
Very fine gold	19637	19,637	Stone flint	2582	2,582
Standard gold	18888	18,888	" paving	2570	2,570
Guinea gold	17793	17,793	" cornelian	2568	2,568
Moidore gold	17140	17,140	" free	2352	2,352
Quicksilver	13600	13,600	Live sulphur	2000	2,000
Lead	11325	11,325	Nitre	1900	1,900
Fine silver	11087	11,087	Alabaster	1875	1,875
Standard silver	10535	10,535	Dry ivory	1815	1,815
Rose copper	9000	9,000	Brimstone	1800	1,800
Copper	8843	8,843	Solid subs. gun.	1745	1,745
Plate brass	8000	8,000	Alum	1714	1,714
Steel	7852	7,852	Ebony	1117	1,117
Cast brass	7850	7,850	Human blood	1054	1,054
Iron	7645	7,645	Amber	1030	1,030
Block-tin	7321	7,321	Cow's milk	1030	1,030
Cast iron	7135	7,135	Sea water	1030	1,030
Lead ore	6800	6,800	Pure water	1000	1,000
Copper ore	3775	3,775	Red wine	993	0,993
Diamond	3400	3,400	Oil of amber	978	0,978
Crystal glass	3150	3,150	Proof spirits	925	0,925
White marble	2707	2,707	Dry oak	925	0,925
Black marble	2704	2,704	Olive oil	913	0,913
Rock crystal	2658	2,658	Loose gunpowd.	872	0,872
Green glass	2620	2,620	Spirits turpen.	864	0,864
Clear glass	2600	2,600	Pure spirits	850	0,850
Brick	2000	2,000	Elm and ash	800	0,800
Oil of turpentine	772	0,772	Sassafras wood	482	0,482
Dry crab-tree	765	0,765	Cork	240	0,240
Ether	732	0,732	Common air	$1\frac{25}{100}$	0,125
White pine	569	0,569	Inflammable air	$0\frac{12}{100}$	0,012

OF THE SPECIFIC GRAVITIES OF BODIES.

The specific gravities of bodies are as their densities or weight, bulk for bulk. A body is said to have two or three times the specific gravity of another, when it contains two or three times as much matter in the same space.

A body immersed in a fluid will sink, if it be heavier than its bulk of the fluid; if it be suspended therein, it will lose so much of what it weighed in the air, as its bulk of the fluid weighs. Hence all bodies of equal bulks, which will sink in fluids, lose equal weights when suspended therein; and unequal bodies lose in proportion to their bulks.

How to find the specific gravities of bodies.

If a body suspended under a scale by a hair, or any other way, at the end of the balance, be first counterpoised in air by a weight in the opposite scale, and then immersed in water, the equilibrium will be destroyed. If then so much weight be put into the scale to which the body is suspended, as will restore the equilibrium, without altering the weights in the opposite scale, that weight which restores the equilibrium will be equal to a quantity of water as big as the immersed body; and if the weight of the body in air be divided by what it loses in water, the quotient will shew how much that body is heavier than its bulk of water. Thus if a guinea, suspended in air, be counterbalanced by 129 grains in the opposite scale, and then, upon being immersed in water, it becomes so much lighter, as to require 7¼ grains to be put into the scale over it, to restore the equilibrium, it shews that a quantity of water, of equal bulk with the guinea, weighs 7,25 grains, by which dividing 129, (the weight of the guinea in air,) the quotient will be 17,793, which shews that the guinea is 17,793 times as heavy as its bulk of water.

Thus may any piece of gold be tried, by weighing it first in air, and then in water; and if by dividing the weight in air by the loss in water, the quotient is 17,793, the gold is good; and if the quotient be 18, or between 18 and 19, the gold is very fine: but if it be less than 17, the gold is too much alloyed by being mixed with some other metal.

If silver be tried in this manner, and found to be 11 times as heavy as water, it is very fine, and if it be 10½ times as heavy, it is standard; but if it be of any less weight, compared with water, it is mixed with some lighter metal, such as tin, &c. If a piece of glass, brass, or lead, be immersed and suspended in different sorts of fluids, the different losses of weights therein will shew how much heavier it is than its bulk of the fluid, that fluid being lightest, in which the immersed body loses least of aerial weight.

Common clear water, for common uses, is generally made a standard for comparing bodies, by which gravity may be represented by unity, or 1; or in case great accuracy be required, by 1,000, where three cyphers are annexed to give room to express the ratios of other gravities in larger numbers in the table. In doing this there is a twofold advantage, the first of which is, that by this means the specific gravities of bodies may be expressed to a greater degree of exactness; and the second, that the numbers of the table, considered as whole numbers, do also express the ounces Avoirdupois contained in a cubic foot of every sort of matter therein specified; because a cubic foot of common water is found to weigh very near 1000 ounces Avoirdupois, or 62½ lbs.

How to try spirituous liquors.

A cubic inch of good brandy, rum, or other proof spirits, weighs 234 grains; therefore if a true cubic inch of any metal weighs 234 grs. less in spirits than in air, it shews that the spirits are proof; and if it loses less of its aerial weight in spirits, they are above proof; but if it lose more, they are under proof; for the better the spirits are, the lighter they are, and the worse the heavier. Or let any fluid of sufficient specific gravity be weighed first in air, next in water, and then in any other liquid. From its weight in air take its weight in water, and the remainder is the weight of its bulk of water. From its weight in air take its weight in the other liquid, and the remainder is the weight of the same quantity of that liquid. Divide the weight of this quantity of liquid by the weight of the same quantity of water, and the quotient will be the specific gravity of the liquid. All bodies expand with heat, and contract with cold, some more, and some less than others; therefore the specific gravities of bodies are not precisely the same in summer as in winter.

EXAMPLES.

1. What proportion of rectified spirits of wine must be mixed with water, to make proof spirits, the specific gravity of rectified spirits being 850, that of proof spirits 925, and that of water 1000?

$$925 \left\{ \begin{array}{l} 1000 = 75 \\ 850 = 75 \end{array} \right. \text{ An equal quantity of each.}$$

2. What proportional weight of rectified spirits of wine and water must be mixed, to make proof spirits, the specific gravities as before?

$1000 \div 850 = 1\frac{3}{17}$; or $\frac{1000}{850} = \frac{20}{17}$, that is, as 20 to 17.

3. What is the specific gravity of best French brandy, consisting of six parts (measure) of rectified spirits of wine, and three parts water?

$850 \times 6 = 5100$ } Then $5100 + 3000 = 8100$, and $6 + 3 = 9$,
$1000 \times 3 = 3000$ } and $8100 \div 9 = 900$, the specific gravity.

4. A storekeeper has 50 gallons of rum, whose specific gravity is 900: how much water must he add to reduce it to standard proof?

$$925 \left\{ \begin{array}{l} 1000 \\ 900 \end{array} \right. \begin{array}{l} 25 \text{ water} \\ 75 \text{ rum} \end{array}$$

As 75 : 25 :: 50 to $16\frac{2}{3}$ gallons of water to be added. Ans.

To find the magnitude of any body, when the weight is known.

RULE. Divide the weight by the specific gravity, and the quotient will be the magnitude sought.

EXAMPLE.

What is the magnitude of several fragments of brick, whose weight is 14 ounces?

$14 \div 2000 = ,007$ of a cubic foot, and $,007 \times 1728 = 12,096$ cubic inches. Ans.

To find the specific gravity of any body, when the magnitude is known.

RULE. Divide the weight by the magnitude, and the quotient will be the specific gravity.

EXAMPLE.

Suppose a piece of ash to contain 8 cubic feet, and weighs 6400 ounces, what is the specific gravity?

$6400 \div 8 = 800$, the specific gravity. Ans.

If the magnitude of any body be multiplied by its specific gravity, the product will be its absolute weight.

EXAMPLE.

What weight of rose copper will be necessary to cover a man of war's bottom, measuring 4000 feet, and the thickness of the copper $\frac{1}{100}$ of a foot?

$\frac{1}{100}$ of 4000 equal 40 cubic feet, and its specific gravity is 9000 ounces Avoirdupois per cubic foot: therefore $40 \times 9000 = 360000$ oz. = 10 tons 0 cwt. 3 qrs. 16 lbs. Ans.

CHRONOLOGICAL CYCLES.

To find the dominical letter for any year, according to the Julian (or old stile) method of calculation.

RULE. Add to the given year its fourth part, and 4, and divide that sum by 7, and if nothing remains, the dominical letter is G; but if there be a remainder, it shews the letter in a retrograde order from G, beginning the reckoning with F, E, &c. Or if it be subtracted from 7, you will have the index of the letter, beginning with A, counting as follows:

A	B	C	D	E	F	G
1	2	3	4	5	6	7

EXAMPLE.

Required the dominical letter for 1827?

1827
456 fourth part, rejecting the remainder.
4

2287÷7=326 and 5 over: then the dominical letter will be B.
Thus 7—5=2, which count, A 1, B 2. Or begin at F, and count backwards, and 5 will come to B; therefore B is the dominical letter for 1827.

To find the dominical letter for any year, according to the new stile, or Gregorian method.

RULE. Divide the year and its fourth part by 7, subtract the remainder, after the division, from 7, and this remainder will be the index of the dominical letter, as before. If nothing remains, it will be G.

EXAMPLE.

Required the dominical letter for 1827?

1827
Fourth part 456
——2283 ÷ 7 = 326 and 1 over; then 7 — 1 = 6.

Therefore F is the dominical letter.

NOTE. Every leap-year has two dominical letters: that found last is the dominical letter from the 25th Feb. to the end of the year; and the next in the order of the alphabet serves from the 1st Jan. to Feb. 24th.

To find the golden number.

RULE. Add 1 to the given year, divide that sum by 19, and the remainder, after the division, will be the golden number: if nothing remains, then 19 will be the golden number.

EXAMPLE.

Required the golden number for 1827?

1827+1=1828÷19=96 and 4 over; therefore 4 is the golden number.

To find the Julian epact, old stile.

RULE. First find the golden number, which multiply by 11, and the product, if less than 30, will be the number required; but if the product exceeds 30, divide it by 30, and the remainder is the epact.

EXAMPLE.

Required the Julian (or old stile) epact for 1827?

Golden number 4×11=44÷30=1 and 14 over; therefore 14 is the epact.

To find the Gregorian (or new stile) epact.

RULE. Subtract 11 from the Julian epact, and if the subtraction cannot be made, add 30, and then subtract, and the remainder will be the Gregorian epact: but if nothing remains, 29 will be the epact.

EXAMPLE.

Required the Gregorian (or new stile) epact for 1827?

Julian epact as above 14—11=3, and 3+30=33—30=3 Greg. epact.

To find the solar or cycle of the sun.

RULE. Add 9 to the given year, divide that sum by 28, and the remainder, after division, is the cycle required: but if nothing remains, the cycle will be 28.

EXAMPLE.

Let it be required to find the solar cycle for 1827?

1827 + 9 = 1836 ÷ 28 = 65 and 16 over; therefore 16 is the solar cycle.

To find the Roman indiction.

RULE. Add 3 to the given year, divide that sum by 15, and the remainder, after division, will be the indiction: but if nothing remains, it will be 15.

EXAMPLE.

Required the year of indiction for 1827?

1827 + 3 = 1830 ÷ 15 = 122 and no remainder: 15 is the Roman indiction.

To find the Julian period.

RULE. Add 4713 to the given year, and the sum will be the Julian period.

EXAMPLE.

What year of the Julian period will answer to 1827?

1827 + 4713 = 6540, the Julian period for 1827.

To find the year of the christian æra, by having the cycle of the sun, the golden number, and Roman indiction.

RULE. Multiply 4845 by the cycle of the sun, 4200 by the golden number, 6916 by the Roman indiction, add the several products together, and divide the sum by 7980, and the remainder, after division, will be the Julian period, from which subtract 4713, and the remainder will be the year of the christian æra.

EXAMPLE.

The cycle of the sun being 16, the golden number 4, and the Roman indiction 15, what year of the christian æra is it?

4845 × 16 = 77520
4200 × 4 = 16800
6916 × 15 = 103740
198060 ÷ 7980 = 24, and 6540 over, which 6540 is the Julian period before found. Now 6540 — 4713 = 1827, the year of the christian æra.

To find the year of the Dionysian period.

RULE. Add to the given year 457, divide that sum by 532, and the remainder will be the number required.

EXAMPLE.

Required the year of the Dionysian period for 1827?

1827+457=2284÷532=4, and 156 over; therefore 156 is the Dionysian period for 1827.

To find the age of the moon on any given day.

RULE. To the given day of the month add the epact, and number of the month: if the sum be less than 30, it is the moon's age; but if it exceeds 30, take 30 from it, and the remainder will be the moon's age.

A Table for the number of the months.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
0	2	1	2	3	4	5	6	8	8	10	10

EXAMPLE.

Required the age of the moon this day, 28th Sept. 1827?

28 day of the month.
3 the epact.
8 number of the month.

39—30=9 days old, Sept. 28th, 1827.

To find the time of the new and full moon, and the first and last quarters.

RULE. Find the moon's age on the given day, and if it be 15, the moon will be full on that day. By counting 7½ days backwards and forwards, you will have the first and last quarters; and by counting backwards and forwards 15 days, you will have the time of the last and next change: but if the age of the moon be greater than 15, take 15 from it, and the remainder will shew how many days have passed since the last full moon. By counting these backwards, you will have the day the last full moon happened on, by knowing which, you can find the change, or either of the quarters, as before. Again, if the age of the moon, on the given day, be less than 15, take that from 15, and the remainder will shew how many days are to come, till the next full moon, which you will have by adding the remainder to the assumed day; and proceeding as before, you will have the days of the change, and either quarters, as above.

EXAMPLE.

For September the 28th, 1827.

First find the moon's age.

28+3+8=39—30=9 days, the age of the moon, as before, being less than 15: then 15—9=6 days the moon has to run till the next full moon.

Proved thus: The assumed day 28 + 6=34 days, from which take the days in the month of September, and you will find the day we will have full moon in the next month. Thus, 34—30=4, bringing the full moon on the 4th October. By adding 7½ days to the last 4 days, (with the odd hours of the 4th Oct.) you will have the last quarter to be on the 12th of Oct. and by taking 7½ days from the 4th Oct. you will have the first quarter on the 27th of Sept. By taking 7½ days from the 27th Sept. you will have new moon on the 20th September, &c.

NOTE. From the change to the full, the moon comes to the south in the afternoon; but from the full to the change, it comes before noon.

A PERPETUAL ALMANACK.

Feb. March Nov.	* Feb. Aug.	May	Jan. Oct.	Jan. * April July	Sept. Dec.	June
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Years.	Days of the week.	Years.	Days of the week.	Years.	Days of the week.
1827	Thursday	1835	Sunday	1843	Wednesday
1828	Saturday*	1836	Tuesday*	1844	Friday*
1829	Sunday	1837	Wednesday	1845	Saturday
1830	Monday	1838	Thursday	1846	Sunday
1831	Tuesday	1839	Friday	1847	Monday
1832	Thursday*	1840	Sunday*	1848	Wednesday*
1833	Friday	1841	Monday	1849	Thursday
1834	Saturday	1842	Tuesday	1850	Friday

The use of the perpetual almanack.

To find on what day of the week any given day in any month will fall.

EXAMPLE.

On what day of the week is the 28th Sept. 1827?

Observe the day of the week annexed to the year 1827, in the column, which you will find to be Thursday; and in the table under the given month, (Sept.) in the sixth row of figures, you will find Thursday to be the 6th, 13th, 20th, and 27th of that month: then count forwards to the 28th, and you will have Friday to be the day required.

Again, what day of the week will next Christmas day fall on? Look as before to the column of 1827, and you have Thursday annexed to it, and then look in the sixth row of figures for December, and you will find that Thursday is the 6th, 13th, 20th, and 27th days of that month: then count backwards, thus, Thursday the 27th, Wednesday the 26th, Tuesday the 25th, which is Christmas day. Therefore Tuesday is the 25th Dec. 1827, and so on.

NOTE. In leap-years, January and February must be taken in the column marked *. Leap-years are marked *.

A Table of the 19 epacts, according to the Julian and Gregorian account, by the golden number.

Golden Number.	Julian Epact.	Greg. Epact.	Golden Number.	Julian Epact.	Greg. Epact.	Golden Number.	Julian Epact.	Greg. Epact.
1	11	29	7	17	6	13	23	12
2	22	11	8	28	17	14	4	23
3	3	22	9	9	28	15	15	4
4	14	3	10	20	9	16	26	15
5	25	14	11	1	20	17	7	26
6	6	25	12	1	18	18	18	7
						19	29	18

A Table of the dominical letters for the new stile, according to the solar cycle, or cycle of the sun.

Cycle.	Dom. Letter.	Cycle.	Dom. Letter.	Cycle.	Dom. Letter.	Cycle.	Dom. Letter.
1§	D C§	8	B	15	G	22	E
2	B	9§	A G§	16	F	23	D
3	A	10	F	17§	E D§	24	C
4	G	11	E	18	C	25§	B A§
5§	F E§	12	D	19	B	26	G
6	D	13§	C B§	20	A	27	F
7	C	14	A	21§	G F§	28	E

NOTE. If you want the epact, first find the golden number for the given year, which you will see in the column headed golden number; and on the same line, to the right hand, you will find either the Julian or Gregorian epact. To find the dominical letter for any given year, first find the cycle of the sun for the required year, which you will find in the column headed cycle; and in the next column, towards the right hand, you will see the dominical letters, or letter, answering thereto. The leap-years are marked § under the cycle and dominical letters.

To find the length of the day and night by the sun's rising and setting.

RULE. Multiply the time the sun rises by 2, and you have the length of the night. Multiply the time the sun sets by 2, and you have the length of the day.

EXAMPLE.

Required the length of the day and night on Sept. 28th, 1827, the sun rising on that day at 7 minutes past 6, and setting at 53 minutes past 5 o'clock?

	h.	*m.*		*h.*	*m.*	
Rises	6	7 × 2 =	12	14		length of the night.
Sets	5	53 × 2 =	11	46		length of the day.
Day and night			24	00	hours.	Ans.

USEFUL TABLES.

Cubic inches contained in pints, quarts, gallons, tierces, hogsheads, puncheons, pipes and tuns of wine.

Cub. inc.	*pt.*						
$28\frac{7}{8}$ =	1	*qt.*					
$57\frac{3}{4}$ =	2 =	1	*gal.*				
231 =	8 =	4 =	1	*tier.*			
9702 =	336 =	168 =	42 = 1	*hhd.*			
14553 =	504 =	252 =	63 = $1\frac{1}{2}$ = 1	*pun.*			
19404 =	672 =	336 =	84 = 2	= $1\frac{1}{3}$ = 1	*pipe.*		
29106 =	1008 =	504 =	126 = 3	= 2	= $1\frac{1}{2}$ = 1	*tun.*	
58212 =	2016 =	1008 =	252 = 6	= 4	= 3	= 2 = 1	

Cubic inches contained in gallons, pecks and bushels of dry measure.

Cub. inc.	*gal.*	
$268\frac{4}{5}$ =	1	*pk.*
$537\frac{3}{5}$ =	2 =	1
$2150\frac{2}{5}$ =	8 =	4 = 1 *bush.*

Inches contained in links, poles, chains, furlongs and miles of long measure.

Inches.	*link.*			
$7\frac{92}{100}$ =	1	*pole.*		
198 =	25 =	1	*cha.*	
792 =	100 =	4 =	1	*fur.*
7920 =	1000 =	40 =	10 =	1
63360 =	8000 =	320 =	80 =	8 = 1 *mile.*

Square inches contained in feet, yards, poles, roods, acres and miles of land measure.

Sq. inches.	*feet.*					
144 =	1	*yard.*				
1296 =	9 =	1	*pole.*			
39204 =	$272\frac{1}{4}$ =	$30\frac{1}{4}$ =	1	*rood.*		
1568160 =	10890 =	1210 =	40 =	1	*acre.*	
6272640 =	43560 =	4840 =	160 =	4 =	1	*m.*
4014489600 =	2787400 =	3097600 =	102400 =	2560 =	640 =	1

Cubic inches contained in pints, quarts and gallons of beer and ale measure.

Cub. inc.	*pt.*	
$35\frac{1}{4}$ =	1	*qt.*
$70\frac{1}{2}$ =	2 =	1
282 =	8 =	4 = 1 *gal.*

OF GAUGING.

I shall not here give the whole art of gauging, there being several books of that art already extant, but will shew how to find the quantity of liquor in any cask, upon occasion.

How to measure any cask.

In order to perform this part of gauging, the three following dimensions of the cask must be truly taken, viz.

The bung diameter, the head diameter, and the length of the cask, all within the cask.

The bunghole must be in the middle of the cask, and the bungstave, and the stave opposite to the bunghole, both regular and even: the heads of the cask equal and truly circular. If so, the distance between the inside of the chine, to the outside of its opposite stave, will be the head diameter within the cask very near.

RULE. Multiply the difference between the bung and head diameter by ,67 or ,64 or ,6 according to the greater or less curvature of the staves, between the head and bung. To the product add the head diameter, and the sum will be a mean diameter; then find the contents of the mean diameter in gallons, which multiply by the length of the cask, and the product is the capacity in wine gallons.

EXAMPLES.

1. Suppose the bung diameter 32 inches, the head diameter 24 inc. and the length 40 inc. required the contents?

Bung diameter 32
Head diameter 24
—8 diff. which multiplied by ,6 = 4,8 + 24, the head diameter, equal 28,8, the mean diameter. Now find the area of the mean diameter by dividing its square by 359,05 for ale, or by 294,12 for wine.

28,8 × 28,8 = 829,44 ÷ 359,05 = 2,31 gallons of ale.
829,44 ÷ 294,12 = 2,71, area in wine.

Then 2,31 × 40, the length, equal 92,4 gallons of ale, the contents.
2,71 × 40, the length, equal 108,4 gallons wine, the contents.

2. Suppose the bung diameter 20 inches, the head diameter 16 inches, and the length 32 inches, required the capacity in wine gallons?

Bung diameter 20
Head diameter 16
—4 diff. then 4 × ,6 = 2,4 + 16, the head diameter, equal 18,4 × 18,4 = 338,56 ÷ 294,12 = 1,15, area of the mean diameter.
Then 1,15 × 32 = 36,8 gallons, the contents.

ULLAGE.

To find the ullage of a cask lying with its axis parallel to the horizon.

Rule. Divide the dry inches of the bung diameter by the bung diameter, to two decimal places, and multiply the quotient thus found by the correspondent multiplier in the following table, and the product, multiplied by the whole contents of the cask, will give the ullage required.

Quot.	,04	,07	,09	,14	,20	,24	,31	,40	,51
Mult.	,2	,3	,4	,5	,6	,7	,8	,9	,1

EXAMPLE.

Suppose the bung diameter 20 inches, of which 5 inches are dry, and the whole contents of the cask 36,8 gallons, required the ullage?

5÷20=,25×,7=,175×36,8=6,44, ullage in gallons.

TO MEASURE A STILL.

Required the contents of a still, the diameter of the greatest section inside being 25 inches, mouth 6 inches, bottom 18 inches, middle section, between the greatest and mouth, 20 inches, middle section, between the greatest and bottom, 24 inches, depth from mouth to the greatest section, 12 inc. and depth from greatest section to the bottom, 18 inches?

Greatest section	25×25= 625	
Mouth	6×6= 36	
Middle, between greatest and mouth	20×20×4=1600	
		2261÷6×12=4522
Greatest section	25×25= 625	
Bottom	18×18= 324	
Middle, between greatest and bottom	24×24×4=2304÷6×18=9759	
		14281

14281×,00344=48½ gallons wine measure.

A KEY TO MENSURATION,

AND

THE PRACTICAL QUESTIONS.

MENSURATION OF SUPERFICIES.

Length 18 feet.
Breadth 9

Ans. 162 feet.

Perpendicular 13,42 feet.
The side 15,5

Ans. 208,010 feet.

One of the longest sides 19,5 feet.
The perpendicular 10,2

Ans. 198,90 feet.

Base 14,1 feet.
Perpen. 12

2)169,2

Ans. 84,6 feet.

Base 14,1
Half perpen. 6

Ans. 84,6

Oblique angled triangle.

Base 15,4 feet.
Half perpen. 3,9

Ans. 60,06 feet.

Of any plain triangle, having the three sides given.

Sides	43,3	4,2
	31,2	16,3
	20,5	27,0
		Differences.

Sum 95,0

Half $47,5 \times 27 \times 16,3 \times 4,2 = 87799,950$

Then the square root of 87799,950 equal 296,31 feet, the answer.

Perpendiculars { 30,1 feet.
24,5

Sum 54,6

Half 27,3
80,5

Ans. 2197,65 feet.

14,6×3=43,8=half sum of the sides; then 12,64, the perpendicular, multiplied by 43,8 equal 553,632 feet, the answer.

Of a dodecagon.

By trigonometry, thus: 360÷12=30 degrees, one of the angles, the half thereof 15 degrees, another angle, whose complement to 90 degrees is 75 degrees, another angle of the dodecagon. Then say, as 15 degrees is to ,5 half the side, so is 75 deg. to the perpendicular, 1,866025. Then 1,866025, multiplied by ,5 half the side, gives the area, 9,330125. Ans.

Of a circle and its parts.

35,5, half the circumference, multiplied by 11,3, half the diameter, gives 401,15, the area, or answer.

As 7 is to 22, so is the diameter to the circumference. Or, as 113 is to 355, so is the diameter to the circumference. Or, as 1 is to 3,141593, so is the diameter to the circumference. Thus,

As 7 : 22 :: 22,6 : 71,028 feet, the circumference.

As 113 : 355 :: 22,6 : 71,000 feet, circumference.

As 1 : 3,141593 :: 22,6 : 71,0000018 feet, the circumference.

As 1 is to ,318309, so is the circumference to the diameter. Or, as 355 is to 113, so is the circumference to the diameter. Or, as 22 is to 7, so is the circumference to the diameter. Thus,

As 1 : ,318309 :: 71 : 22,599939 feet, the diameter.

As 355 : 113 :: 71 : 22,6 feet, the diameter.

As 22 : 7 :: 71 : 22,59+ feet, the diameter.

As 1, (the square of the diameter,) is to ,7854, so is 510,76, (the square of 22,6, the diameter of the given circle,) to 401,15 ft. the area, by Van Culen. Or, as 452 : 355 :: 510,76 to 401,15 feet, the area, as before, by Metius. Or, as 14 : 11 :: 510,76 to 401,31 ft. the area, by Archimedes' proportion.

355×4=1420; then say, as 1420 : 113 :: 5041, (the square of the circumference,) to 401,15 feet, the area.

As 1 : ,8862 :: 22,6 : 20,02812 feet, the answer.

,8862 is equal to the side of a square, when the diameter is 1.

As 1 : ,2821 :: 71 : 20,0291 feet, answer.

If the circumference of any circle be 1, the side of the square equal will be ,2821.

As 1 : ,7071 : : 22,6 : 15,98046 feet, answer.

If the diameter of a circle be 1, the side of the square inscribed will be ,7071.

As 1 : ,2251 : : 71 : 15,9821 feet, answer.

If the circumference be 1, the side of the square inscribed will be ,2251.

As 1 : 1,2732 : : 401,15 : 510,744180, equal to the square of the diameter, whose square root is 22,599 feet, the answer.

If the area of a circle be 1, the square of the diameter thereof will be 1,2732.

As 1 : 12,56637 : : 401,15 : 5040,9999255, whose square root is 70,999 + feet, answer.

If the area of a circle be 1, the square of the circumference will be 12,56637.

As 1 : 401,15 : : ,6366 : 255,372090, whose square root is 15,98 feet, answer.

If the area of a circle be 1, the area of a square inscribed within that circle will be ,6366.

As 1 : 1,4142 : : 15,98 : 22,598916, the diameter, answer.

If the side of a square be 1, the diameter of a circle that will circumscribe that square, will be 1,4142.

As 1 : 1,128 : : 90,0291 : 22,5928848 feet, the answer.

If the side of a square be 1, the diameter of a circle equal thereunto will be 1,128.

As 1 : 4,443 : : 15,98 : 70,99014 feet, the answer.

If the side of a square be 1, the circumference of a circle that will encompass that square, will be 4,443.

As 1 : 3,545 : : 90,0291 : 71,0031595 feet, answer.

If the side of a square be 1, the circumference of a circle that will be equal thereunto, shall be 3,545.

Of a semicircle.

17,75, half the circumference, multiplied by 11,3, half the diameter, gives 200,575 feet, the area. Ans.

11,3 the semidiameter.
8,875 half the circumference.

100,2875 feet, the area. Ans.

24,5 semidiameter.
22,8 half the arch line.

558,6 feet, the area. Ans.

17,5 half the chord line.
17,5

306,25÷9,6=31,9+9,6=41,5, the diameter of the circle.
41,5÷2=20,75, the semidiameter, less 9,6 equal 11,15, the perpendicular, which multiplied by 17,5, half the chord line, equal 195,125, the area of the triangle. Then + 306,25 the square of 9,6=398,41, sum. √398,41=19,96×8=159,68—35=124,68÷3=41,56÷2=20,78×20,75, semidiameter, equal 431,1850, area of the sector, from which take 195,125, area of the triangle, leaving 236,060, area of the segment. Ans.

61,6 the transverse diameter.
44,4 the conjugate diameter.

2735,04 the rectangle, multiplied by ,7854=2148,100416 feet, the area.

53,75×39,25=2109,6875×2=4219,3750÷3=1406,4583 feet, area.

OF SOLIDS.

17,5×17,5×17,5 equal 5359,375 inches; or 3 ft. 175 ins. Ans.

21×21 equal 441×15 ft. the length, and the product is 6615÷144 equal 45,9+ feet, the solidity. Ans.

13,51 the perpendicular.
7,3 half the base.

105,378 the area of the base, multiplied by 19,5 feet, the length, gives 2054,8710÷144=14,27 feet, the solidity. Ans.

18,5×18,5 equal 342,25, area of the base, which multiplied by 5, one-third of the altitude, equal 1711,25 inches, which divide by 144, and you will have 11,88 feet, the solidity. Ans.

21,5×21,5 equal 462,25, the rectangle, which multiplied by ,7854, equal 363,05115, area of the base, which multiplied by 16 feet, the length, equal 5808,8164 inches, which divide by 144, and the quotient is 40,34 feet. Ans.

26,5×26,5 equal 702,25, which multiplied by ,7854, equal 551,64, the area of the base, which multiplied by 5,5, a third part of 16,5, the product is 3033,47÷144, and the quotient is 21,06 feet. Ans.

18×18 equal 324 12×12 equal 144

18×12 equal 216; then 216+324+144 equal 684, which multiply by 6, a third part of 18, and the product is 4104, which divide by 144, and the quotient is 28,5 feet. Ans.

18×18 equal 324 9×9 equal 81

18×9 equal 162; then 324+81+162 equal 567, which multiply by ,7854, and the product is 445,3218÷144 equal 3,09 feet, the triple area of a mean base, which multiplied by 4,75, a third of 14,25, the product is 14,6775 feet, the solidity. Ans.

38 add 15 equal 53 multiply 16 equal 848 reserve.
30 add 19 equal 49 multiply 12 equal 588

1436 sum, which multiplied by 2, a third of 6, the product is 2872, which divide by 144, and the quotient will be 19,94 feet. Ans.

44 add 13 equal 57, sum, which multiplied by 14 equal 798, reserved. 26 add 22 equal 48, sum, which multiplied by 26 equal 1248, add 798 equal 2046, which multiplied by ,7854 equal 1606,9284, which multiplied by 3 equal 4820,7852, which divide by 144, and the quotient will be 33,47 feet, the answer.

20 multiply 20 multiply 20 equal 8000, multiply 11 equal 88000, which divided by 21 equal 4190,47 inches, or 2,424 feet. Ans.

33 multiply 33 equal 1089, multiply 55 equal 59895, multiply ,5236 equal 31361,0220 inches, the solidity. Ans.

36 multiply 36 equal 1296, multiply ,7854 equal 1017,8784, which multiplied by 33 equal 33589,9272, divided by 2 equal 16794,9936, divided by 1728, the quotient is 9,71 feet, the answer.

36 multiply 36 equal 1296, multiply ,41888 equal 542,88848, multiply 99 equal 53743,07952, divide by 1728, and the quotient is 31,10184 feet. Ans.

Of carpenters' work.

47,75 multiply 53,5 equal 2554,625 feet decimally, which divide by 10, and the quotient is 25 squares, and 54 feet, the answer.

12,25 multiply 82,5 equal 1010,625 feet decimally, which divided by 10, equal 10 squares and 10 feet, the answer.

44,5 multiply 18,25 equal 812,125 the flat.
406,62 half flat.

10)12,18 flat and half flat.

Ans. 12 squares 18 feet.

Bricklayers' work.

72,5 multiply 19,25 equal 1395,625 multiply 11, (the number of halves in 5½ bricks,) equal 15351,875, which divided by 3, equal 5117,291, which divided by 272,25, equal 18 rods 3 quar. 12 feet. Ans.

245,75 multiply 16,5 equal 4054,875, multiply 5, (the number of halves in 2½ bricks,) equal 20270, rejecting decimals: then 20270 divided by 3, equal 6756, which divided by 272, equal 24 rods 3 quar. 24 feet. Ans.

To find how many bricks are necessary to build a house of given dimensions.

8 multiply 4 multiply 2,5 equal 80 solid inches in a brick: then 1728 divided by 80, equal 21,6 bricks in a solid foot. 44 add 44 add 40 add 40 equal 168 less 4 equal 164, which multiplied by 20, (the height,) equal 3280, which multiplied by 21,6, (the bricks in a solid foot,) equal 70848 bricks,

18,75 multiply 12,5 equal 234,375, the first story. 14,5 multiply 9 equal 130,5, the second story. 10,25 multiply 7 equal 71,75, the third story. 13,75 multiply 6,5 equal 89,375, the shaft. 1,25 multiply 12 equal 15, the fetter. These added all together, the sum is 541 feet, which doubled is 1082, and this divided by 272, equal 3 rods 3 quar. 62 feet. Ans.

Plasterers' work.

59,75 multiply 24,5 equal 1463,875 feet, which divided by 9, the quotient is 162,65 yards, the answer.

141,5 multiply 11,25 equal 1591,875, which divided by 9, equal 176,87 yards, the answer.

Joiners' work.

126,25 multiply 15,75 equal 1988,4375, which divided by 9, equal 220,8 yards, the answer.

Masons' work.

97,417 multiply 18,25 multiply 2,25 equal 4000 feet 2 inches. Ans.

107,75 multiply 20,5 equal 2208,875 superficial feet, the answer.

Of board measure.

16 multiply 13 equal 208, which divided by 12, equal 17 ft. 4 ine. answer. Or, 156, (the inches in 13 feet,) multiplied by 16, equal 2496, which divide by 144, and the quotient is 17 feet 4 inches, the answer. Or say,

As 12 : 13 :: 16 : 17 ft. 4 ine. Ans.

Of squared timber.

15 multiply 15 multiply 18 equal 4050, which divide by 144, and the quotient is 28 feet 1½ inches, the answer.

33 multiply 19 multiply 16,75 equal 10502,25, which divided by 144, equal 72 feet 11 inches, answer, the true way.

33 add 19 equal 52, sum, and half thereof 26, which multiplied by 26, equal 676: then 676 multiplied by 16,75, the length, is 11323, which divide by 144, and the quotient is 78,63 feet, answer, the false way.

The true content is 72,93 feet, and the false content 78,63 feet, too much by 5,70 feet.

18 multiply 18 equal 324; then 1728 divided by 324, equal 5⅓ ine. Ans.

22 multiply 15 equal 330; then 1728 divided by 330, equal 5,23 ine. Ans.

Of unequal squared timber.

25 add 9 equal 34, sum, and half thereof is 17, the side of the square in the middle: then 17 multiply 17 multiply 20 equal 5780, which divide by 144, and the quotient will be 40,13 feet, answer, the false way.

25 multiply 9 equal 225, and 25 less 9 equal 16, difference of the sides: then 16 multiply 16 equal 256, which divided by 3, equal 85,333: then 85,333 add 225 equal 310,333, which multiplied by 20, equal 6206,660, which divide by 144, and the quotient is 43,101 feet, the true way: so the difference is 3 feet nearly.

36 multiply 20 equal 640, and 10 multiply 6 equal 60, and 640 multiply 60 equal 38400, the square root whereof is 195,959, the mean proportional: then add 640, the greater base, and 60, the lesser base, to the mean proportional, and the sum will be 895,959; which multiply by 6, a third of the length, and the product will be 5375,754, which divide by 144, and the quotient is 37,33 feet, answer, the true way.

32 add 10 equal 42, sum, and 20 add 6 equal 26, sum: then half of 42 is 21, and half of 26 is 13: then 21 multiply 13 equal 273, area in the middle: then 273 multiply 18, the length, and the product is 4914, which divide by 144, and the quotient will be 34,12 feet, answer, the false way.

Of round timber, whose bases are equal.

96 divided by 4 equal 24, and 24 multiply 24 equal 576, area of the base: then 576 multiply 18, the length, and the product is 10368, which divide by 144, and the quotient will be 72 feet, answer, the false way.

96 multiply 96 equal 9216, multiply ,07958 equal 733,40928, the area: then 733,40928 multiply 18 equal 13201,36704, which divide by 144, and the quotient will be 91,67 feet, answer, the true way.

86 divided by 4 equal 21,5, and 21,5 multiply 21,5 equal 462,25, area: then 462,25 multiply 20 equal 9245,00, which divide by 144, and the quotient will be 64,2 feet, answer, the false way.

86 multiply 86 equal 7396, multiply ,07958 equal 588,57368, area: then 588,57368 multiply 20 equal 11771,47360, which divide by 144, and the quotient will be 81,74 feet, answer, the true way.

Of round timber, whose bases are unequal.

36 multiply 9 equal 324, the rest. then 36 less 9 equal 27, diff. then 27 multiply 27 equal 729, which divided by 3 equal 243, one-third: then 324 add 243 equal 567, multiply ,7854 equal 445,3218, a mean area: then 445,3218 multiply 24 equal 10687,7232, which divide by 144, and the quotient will be 74,22 feet, the answer.

136 multiply 32 equal 4352, rest. then 136 less 32 equal 104, diff. then 104 multiply 104 equal 10816, which divided by 3 equal 3605,333, one-third: then 4352 add 3605,333 equal 7957,333, a mean cir. squared: then 7957,333 multiply ,07958 equal 633,24456014, the mean area: then 633,24456014 multiply 21 equal 13298,1376294, which divide by 144, and the quotient will be 92,34 feet, the answer.

OF THE FIVE REGULAR BODIES.

1st. *Of a tetraedron.*

10,392 is the perpendicular composing the body; then 10,392 multiply 6, (half the side.) equal 62,352, the area of the triangle. One-third of 62,352 is 20,784; then 20,784 multiply 9,798, (the perpendicular height,) equal 203,641632 inches, the answer.

2nd. *Of an octaedron.*

5,6568, a third part of its length: 12 multiply 12 equal 144, the area of the square base: then 5,6568 multiply 144 equal 814,5792, the solidity. Ans.

5d. *Of a dodecaedron.*

13,36219 is the altitude of one of the pentangled pyramids, and 8,258292, the perpendicular of the pentagon: then 8,258292 multiply 30, (half the sum of the sides,) equal 247,748760, the area of the pentagon: then 247,748760 multiply 13,362192 equal 1103,48783, content of one pyramid, which multiply by 12, and the product will be 13241,85392, the solidity. Ans.

4th. *Of an icosaedron.*

10,39224, the perpendicular of the triangle, multiplied by 6, half the side, is 62,35344, which multiplied by 20 equal 1247,06880: then 62,35344 multiply 3,0230456, (a third part of the altitude of the pyramid,) = 188,497292, multiply 20, (the number of triangles,) and the product will be 3769,945840, the solidity. Ans.

By the tabular numbers.

7,663119, tabular number, multiplied by 1728, the cube of the side, the product will be 13241,869632, the solid content, nearly as before. Ans.

29,645729, tabular number, multiplied by 144, the square of the side, the product will be 2972,984976, the superficial content. Ans.

,4714045, tabular number, multiplied by 8000, the cube of the side, the product will be 3771,2360000, the solid content. Ans.

3,464102, tabular number, multiplied by 400, the square of the side, the product will be 1385,640800, the superficial content. Ans.

To measure an irregular body.

32 multiply 32 equal 1024, multiply ,7854 equal 804,2496, the area of the base: then 804,2496 multiply 14, (the fall of the water,) equal 11259,49, which divide by 1728, (the solid inches in a foot,) and the quotient will be 6,51 feet, the solidity. Ans.

PRACTICAL QUESTIONS.

Question 1.

47,75 multiply 18,5 equal 883,375, which divide by 9, and the quotient will be 98 yards 1 foot, the answer.

Question 2.

21,5 multiply 17,5 equal 376,25; then 1,5 multiply 1,5 equal 2,25, the area of one stone. 376,25 divided by 2,25, the quotient is 167 stones. Ans.

Question 3.

109,75 multiply 92,5 equal 1015,1875, the content of the room: then 5,75 multiply 6,5 equal 37,375, multiply 2 equal 74,750, the content of the windows; and 1015,1875 less 74,750 equal 940,4375: then 3,75 multiply 3 equal 11,25, (the square feet in a yard of tapestry,) which divided thus, 940,4375 ÷ 11,25 = 83,60 yards, the answer.

Question 4.

3,1416 multiply 27,5 equal 86,394, the circumference; then 86,394 multiply 27,5, (the diameter,) equal 2375,8350, which divide by 144, and the quotient will be 16,49 feet, the superficial content. Ans.

Then 2375,8350, (inches in the superficial content,) divided by 6, the quotient is 395,9725, which multiplied by 27,5 equal 10889,24375, which divide by 1728, and the quotient will be 6,3 feet, the solid content. Ans.

Question 5.

24 multiply 24 equal 576, and 576 multiply ,7854 equal 452,3904, and 400 multiply ,7854 equal 314,1600, and 20 multiply 20 equal 400: then 314,1600 add 452,3904 equal 766,5504, the curve superficies, to which add 452,3904, the base, and the sum will be 1218,9408, the superficial content. Ans.

12 multiply 12 equal 144, multiply 3 equal 432, add 100, (the square of the altitude,) equal 532, which multiplied by 10, (the altitude,) equal 5320, multiply ,5236 equal 2785,5520, the solidity in inches. Ans.

Question 6.

18,5 divided by 4 equal 4,625; then 4,625 multiply 4,625 multiply 24, and divide by 40, equal 12 tons 33 feet 4½ inches. Ans.

Question 7.

33	7		the length.
×18	9		the breadth.
629	8	3	parts.
×5	9	0	the depth.
3620	8		÷324 and the quotient is 11 floors 56 feet, the answer.

By cross multiplication.
18×18=324 feet in a floor of earth.

Question 8.

48,75 multiply 35,5 equal 1730+, (the decimals of a foot not noticed:) then 1730 divided by 100, the quotient will be 17 squares 30 feet. Ans.

Question 9.

42	1728	94
42	2	94
1764	3456	8836 multiply 94 equal 830584, the cube.

1764 multiply ,7854 equal 1385,4456, multiply 94 equal 130231,8864÷3 and the quotient will be 43410,6288: then say, as 43410,6288 : 830584 :: 3456 to 66124, the cube of the length; and the cube root of 66124 is 40,43 inches, the length upon the perpendicular. Ans.

If it had been 3 feet, the length would be 46,29 inches.

All the solid bodies are in triplicate reason of their homologous sides, by Euclid, 12, 12; 12, 18; and 11, 33.

Question 10.

21 less 3 equal 18, difference; then say, to find the length of the whole pyramid, as 18 : 12 :: 21 to 14, the length of the whole pyramid 14 feet, or 168 inches: then the solid content of the whole pyramid is 24696 inches: then 1728 multiply 5 equal 8640 inches. the solid content of 5 feet: then 24696 less 8640 equal 16056 inches: then 168×168×168=4741632. Then say,

As 24696 : 4741632 :: 16056 to 3082752, whose cube root is 145,54. Subtract this root from 168. the length, and there will remain 22,46 inches,

Question 11.

All circles are in duplicate reason of their diameters, by Euclid 12, 2.

20 multiply 20 equal 400; then say, as 20 : 400 :: 9 to 180, equal to the square of the semidiameter of the circle belonging to the first man: then as 20 : 400 :: 6 to 120, equal to the square of the semidiameter of the circle belonging to the second man. Again say, as 20 : 400 :: 5 to 100, equal to the square of the semidiameter of the circle belonging to the third man: then 400 less 180 equal 220, whose square root is 14,83 inches, which subtract from 20 inches, (the semidiameter of the whole stone,) and there remains 5,17 inches, the breadth of the ring, or part of the stone, which must be ground down by the first man. Then 220 less 120 equal 100, whose square root is 10; then 14,83 less 10 equal 4,83 inches, the breadth or part of the stone to be ground down by the second man. Then 5,17 inches by the first man, and 4,83 inches by the second man, make 10 inches, which taken from 20 inches, (the semidiameter of the whole stone,) there remains 10 inches, the breadth or ring belonging to the third man.

Question 12.

12×12×2×20÷144=40 feet, the solid content. Then say,

As 1½ : 40 :: 12 : 384 feet of boards, the answer.

Question 13.

11×11×2×20÷144=33,6+ the solidity, when hewn square. Ans.

Question 14.

All such questions as this admit of no work, but cut it at one-third of its altitude.

Question 15.

First suppose 3 numbers, according to the given proportion, and suppose the depth 7 feet; then the width will be 10 feet, and the length 42 feet: then 7×10×42=2940, and 567,5×1728=635040 inc. then 10×10×10, (the width,)=1000. Then say, as 2940 : 1000 :: 635040 to 216000, the cube of the true width, whose cube root is 60, and seven-tenths thereof 42, the true depth; and 6 times 42 is 252 inches, the true length. Then 60 multiply 42 multiply 252 equal 635040÷2150,42, (the solid inches in a bushel,) and the quotient will be 295+ bushels, the answer.

Question 16.

18,5 multiply 18,5 equal 341,25, multiply ,7854 equal 268,803150, multiply 8 equal 2150,425200, the solid inches in a bushel: then 2150,4252÷7,5= 286,72336÷,7854=365,0666, whose square root is 19,107 inc. the diameter, when the depth is 7,5 inches, the answer.

Question 17.

660 feet, the length of an acre.
66 feet, the breadth of an acre.

43560 the square in an acre; then say, as 1 : 1,2732 :: 43560 to 55460,5920, whose square root is 235,5, (diameter of an acre, when in a circle,) ÷ 2 = 117,75, half the diameter: therefore the cord must be 117 feet 9 inches long, the answer.

Question 18.

16,5 multiply 16,5 equal 272,25, the area of the old crib: then 272,25 multiply 3 equal 816,75, whose square root is 28,57, or 28 feet 7 inches nearly, the answer.

Question 19.

20 multiply 20 multiply 20 equal 8000; then say, (by Euclid, 11, 33,) as 217 : 8000 :: 1000 to 36870, whose cube root is 33,28 inches, the length.

Then say, as 20 : 33,28 :: 15 : 24,96 breadth.

20 : 33,28 :: 8 : 13,312 thickness.

Question 20.

4 multiply 4 multiply 4 equal 64, and 9 multiply 9 multiply 9 equal 729: then say, (by Euclid, 12, 18,) as 64 : 9 :: 729 to 102,515 lbs. or 102 lbs. 8 oz. 4 drs. the answer.

Question 21.

5 multiply 5 multiply 5 equal 125, and 30 multiply 30 multiply 30 equal 27000: then say, (by Euclid, 12, 12,) as 125 : 12,25 :: 27000 to 2646 lbs. the answer.

Question 22.

6 multiply 6 multiply 6 equal 216; then say, as 11 : 216 :: 500 to 981,1818, whose cube root is 21,4 inches, the diameter. Ans.

Question 23.

4,2284 the tabular number. 12 multiply 12, divide by 3, equal 48.

257 the square of a mean side. 21 multiply 9, add 48, equal 257.

1144,3308 × 15 ÷ 144 = 119,2 solid feet, the answer.

Question 24.

First find the length of the whole cone, thus: 36 less 20 equal 16; then say, as 16 : 215 :: 36 to 483,75 inches, the whole length of the cone; and then find the content of the whole cone, thus: 36 multiply 36 multiply ,7854 equal 1017,8784, area of the base: then 1017,8784 multiply 52,161 equal 164132,88 ÷ 1728 = 94,98 feet, the length of the whole cone.

Then find the content of the top-part, that is wanting: ,7854, the area of unity: 20 multiply 20 equal 400, multiply ,7854 equal 314,1600, (area of the lesser base,) ÷ 3 = 104,72, a third part: then 104,72 × 268,75, (altitude of the top-part,) ÷ 1728 = 16,28 feet, content of the top-part.

Then from 94,98 feet, the content of the whole cone, take 16,28 feet, the content of the top-piece, and there remains 78,70 ft. content of the frustum.

Question 25.

200 multiply 200 multiply 200 equal 8000000; then 159610 ÷ 26171 = 185781, the sum: then say, as 26171 : 8000000 :: 185781 to 56789881, whose cube root is 384.3 inches, the length of the whole cone.

Then, to find the diameter of the lesser base, 26171 ÷ 200 × 3 = 392,565, area of the lesser base: then by problem 10, (the proportion of a circle,) in mensuration, say, as 1 : 1,2732 :: 392,565 to 499,8137580, whose square root is 22,35: then say, as 200 : 22,35 :: 384,3 to 42,94, greater diameter, and the lesser diameter 22,35.

Question 26.

5 multiply 5 equal 25, and 5 multiply 2 equal 10, and 2 multiply 2 equal 4: then 25 add 10 add 4 equal 39, the sum. 12 ÷ 3 = 4, and 20 ÷ 4 = 5 feet, the triple of a mean area: then say, as 1 : 1,27324 :: 5 to 6,3662, the triple square of a mean diameter. Then say, as 39 : 6,3662 :: 25 to 4,080897, the square of the greater diameter, whose square root is 2,020123 feet, which is 24,24147 inches, the greater diameter. Then, to find the lesser, say,

As 5 : 24,24147 :: 2 to 9,69659 inches, the lesser diameter.

Question 27.

Suppose the breadth 2 feet, then the height must be 10 feet, and length 80: then 2 multiply 10 multiply 80 equal 1600: then 2 multiply 2 multiply 2 equal 8: then say, as 1600 : 8 :: 18225 to 91,125, whose cube root is 4,5, which is the breadth. Then 4,5 multiply 5 equal 22,5, the height, and 22,5 multiply 8 equal 180, the length.

Question 28.

39 multiply 39 equal 1521, and 15 multiply 15 equal 225: then 1521 less 225 equal 1296, whose square root is 36 feet, the piece standing, and the piece broken off 39 feet, both which added together make 75 feet, the whole length of the pole. Ans.

Question 29.

(By Euclid, 6, 4.) As 4,1 : 5 :: 65 to 79,26 feet. Ans.

Question 30.

The solidity and superficies is as 1 to 6: when the axis of the globe is 1, then say, as 18 : 8 :: 6 to $2\frac{2}{3}$, answer for No. 31.

Question 31.

If the diameter be 1, the solidity will be ,5236, and the superficies 3,1416, that is, as 1 to 6: and to find the superficial content, multiply 3,1416 by the square of the axis, and the product is the superficial content. For the solidity, multiply ,5236 by the cube of the axis, and the product is the solid content. Therefore, because ,5236 is a sixth part of 3,1416, you must take 6 for the diameter sought. Therefore 6 is the answer for No. 30.

Question 32.

RULE. From the square of the length of the pole, (that is, of the sum of the hypothenuse and perpendicular,) take the square of the base, and divide the remainder by twice the length of the pole, and the quotient will be the perpendicular height, where it was cut off.

$$\frac{75\times75-55\times55}{75\times2} = 17\frac{1}{2} \text{ feet, answer.}$$

Question 33.

16 multiply 16 multiply 16 equal 4096; then say, as 1 : ,5236 :: 4096 to 2144,6656 ÷ 2 = 1072,3328, which is the solidity of the metal of the greater shell, as also the concavity. Then say, as ,5236 : 1 :: 1072,3328 to 2048, whose cube root is 12,699, which is the diameter of the second shell. Then 7 add 5 equal 12; and say, as 12 : 5 :: 1072,3328 to 446,805, the solidity of the concavity of the second shell. Then say, as ,5236 : 1 :: 446,805 to 853,333, whose cube root is 9,485, the diameter of the least shell. Then 9 add 4 equal 13; and say, as 13 : 4 :: 446,805 to 137,47846, the solid content of the concavity of the third shell. Then say, as ,5236 : 1 :: 137,47846 to 262,5639, whose cube root is 6,4034, the diameter of the least shell's concavity. Then 16, the diameter of the greatest, less 12,699, the diameter of the second, equal 3,301 ÷ 2 = 1,65, the thickness of the metal of the greatest. Then from 12,699, the diameter of the second, take 9,485, the diameter of the least, and there remains 3,314 ÷ 2 = 1,657, the thickness of metal of the second. Then from 9,485, the diameter of the least, take 6,403, the diameter of the concavity, and there remains 3,082 ÷ 2 = 1,541, the thickness of metal of the least.

The metal of the greatest is 1072,33 solid inches, which divide by 4, (for every 4 solid inches is one-fourth of a pound,) and the quotient is 268,08 lbs.

The metal of the second is 623,82 solid inches, which divide by 4, and the quotient is 156,38 lbs. the weight of the second. The metal of the least is 309,32 solid inches, which divide by 4, and the quotient is 77,33 lbs. the weight of the least shell.

The diameter of the second shell is 12,699 inches, and of the least shell, 9,485 inches. The thickness of the metal of the greatest is 1,65 inc. of the second, 1,607 inc. and of the third, 1,541 inc. The weight of the greatest is 268,08 lbs. of the second, 156,38 lbs. and of the least, 77,33 lbs.

Question 34.

$$\frac{7,5 \times 2,95 \times 2}{3,75} = 9 \text{ inches. Ans.}$$

Question 35.

$18 \times 14 = 252 \div 3 = 84 + 8 = 10\frac{1}{2}$ inches. Ans.

Question 36.

1296 inches make a square yard, and 16 feet 5 inches equal 197 inc. then 1296 divided by 197, equal $6\frac{114}{197}$ inc. from the edge. Ans.

Question 37.

The square root of 2500 equal 50 each side; then 2500 divided by 2, equal 1250, the square root whereof is 35,35: then 50 less 35,35 equal 14,65 yards, the breadth of the walk. Ans.

Question 38.

20 multiply 20 equal 400, and 10 multiply 10 equal 100, and again, 10 multiply 10 equal 100: then 100 add 100 equal 200, the two floors; and 400 less 200 equal 200 feet. Ans.

Question 39.

12 multiply 12 multiply 6 equal 864, a solid half foot.
6 multiply 6 multiply 6 equal 216, half a foot solid.

Ans. 648 inc.

Question 40.

$2 \times 3 \times 4 \times 5 \times 6 = 720 + 1 = 721 \div 7 = 103$ peaches. Ans.

Question 41.

The dog runs 16 miles an hour, and the hare 10 miles, the dog gaining 6 miles in running 1 hour: then say, as 1 : 10 :: 45 to 660 feet; and 12 rods added to 660 feet, make 858 feet, the distance the hare was when the dog started. Then say, as 6 : 858 :: 16 to 2288 feet, run by the dog. Now, as 16 : 1 :: 2288 to $97\frac{1}{2}$ seconds, or 1 min. $37\frac{1}{2}$ sec. the length of the chase; and the length of ground run by the dog, was 2288 feet, or 3 fur. $18\frac{3}{5}$ rods.

Question 42.

6 hours in the forenoon, and 3 in the afternoon, make 9 hours, the messenger from Washington was before the other: then 9 multiply 4 equal 36 miles he travelled, before the other messenger started. Then 7 add 4 equal 11 miles, both lessen the road per hour. Then 100 miles, (the distance between both places,) less 36 equal 64. Then say, as 11 : 4 :: 64 to $23\frac{3}{11}$ miles, (the messenger from Washington travelled, after the other from Pittsburgh started,) add 36, (he travelled before he left Pittsburgh,) equal $59\frac{3}{11}$ miles, from Washington, where they meet, and $40\frac{8}{11}$ miles from Pittsburgh, that is, 100 less $59\frac{3}{8}$ equal $40\frac{8}{11}$. Ans.

Question 43.

As 22 : 2 :: 1 to $\frac{2}{22}$, or $\frac{1}{11}$. As 24 : 3 :: 1 to $\frac{3}{24}$. Then $\frac{1}{11}-\frac{3}{24}=\frac{1}{37\frac{1}{3}}$. Then say, as $1 : \frac{1}{37\frac{1}{3}} :: 22$ to $1\frac{11}{187}$. Then as $1\frac{11}{187} : 1 :: \frac{1}{1}$ to 17, the number of times they go round the field, before B overtakes A. Ans.

Question 44.

As 5 : 7 :: 60 to 84 gallons in 1 hour. As 17 : 20 :: 60 to $70\frac{10}{17}$ gallons in 1 hour. Then 84 gallons, (filled in 1 hr.) less $70\frac{10}{17}$ gallons, (emptied in 1 hour,) equal $13\frac{7}{17}$ gallons, (difference in 1 hour.) $13\frac{7}{17} \times 2$ hrs. $= 26\frac{14}{17}$ gallons. Then $100-26\frac{14}{17}=73\frac{3}{17}$ galls. remaining to be filled. Then say,

As 7 : 5 :: $73\frac{3}{17}$ to $51\frac{11}{17}$ min. past 6 o'clock. Ans.

Question 45.

As $\frac{3}{8}$: 1800 :: $\frac{1}{1}$ to 4800 dollars. Then $4800+6000=10800=\frac{4}{7}$ of his fortune. Then say,

As $\frac{4}{7}$: 10800 :: $\frac{1}{1}$ to 15120 dollars. Ans.

Question 46.

$12\times10=120$, whose square root is 10,954 lbs. Ans.

Question 47.

$\frac{4}{5}$ of $\frac{2}{3}=\frac{10}{40}=\frac{1}{4}$. Then say, as $\frac{1}{4}$: 1500 :: $\frac{1}{1}$ to $6000. Ans.

Question 48.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=1\frac{1}{2}$. Then $1-1\frac{1}{2}=\frac{1}{1\frac{1}{2}}$; therefore $\frac{1}{1\frac{1}{2}}$ is equal to $50+30=80$. Then as $\frac{1}{1\frac{1}{2}} : 80 :: \frac{1}{1}$ to 960, number of trees.

Question 49.

Suppose A and B had 50 dollars: then say, as 5 : 7 :: 50 to 70, B's and C's: then say, as 7 : 1 :: 70 to 10, C's—B's. Then $70-10=60\div2=30$ for B; and $30-10=20$ for A. Then $30+10=40$ for C. Then it will be, A 20 + B 30 + C 40 = 90. Then say,

As 90 : 200 :: 20 to $44\frac{4}{9}$ for A.
As 90 : 200 :: 30 to $66\frac{6}{9}$ for B.
As 90 : 200 :: 40 to $88\frac{8}{9}$ for C.

Question 50.

$112,8\div12=9,4=$ the second number: then as 4 : 9,4 :: 12 to 28,2=fourth number: then 28,2—9,4=18,8, difference of the second and fourth. Ans.

Question 51.

$40\times7=280$, and $40\times8=320$, and $40\times9=360$: then A will travel 280 miles, B 320, and C 360: then $280\div7$, and $320\div8$, and $360\div9=40$ days, after they started. Ans.

Question 52.

$$\frac{1}{2}+\frac{1}{3}=\frac{1\times7+3\times5}{5\times7}=\frac{22}{35}\text{ Ans.}$$

Question 53.

A + B + C + D = 20000 dolls.
B + C + D + E = 26400
A + C + D + E = 24000
A + B + C + E = 22400
A + B + D + E = 25600

118400 ÷ 4, (the number of combinations,) equal 29600 dollars.

Sum of their fortunes, 29600 dolls.
B + C + D + E = 26400
——— 3200, first daughter's fortune.

Sum of their fortunes, 29600
A + C + D + E = 24000
——— 5600, Charlotte, 2nd daughter.

A + B + C + D + E = 29600
A + B + D + E = 25600
——— 4000, the 3d, or C's fortune.

A + B + C + D + E = 29600
A + B + C + E = 22400
——— 7200, the 4th, or D's fortune.

A + B + C + D + E = 29600
A + B + C + D = 20000
——— 9600, the 5th, or E's fortune.

Question 54.

5½ leagues equal 16,5 miles; then 16,5 — $\sqrt{42056462}$ + 6 × 6 = 13,948 miles, equal to 73619 feet nearly: then 73619 × 73169 ÷ 42056462 = 129 feet nearly. Ans.

Question 55.

Suppose A's	100	
B's is ⅔	66⅔	As 216⅔ : 234 :: 100 to 108 A's
C's is ½	50	As 216⅔ : 234 :: 66⅔ to 72 B's
	216⅔	As 216⅔ : 234 :: 50 to 54 C's

Question 56.

Suppose $16 the first time, 48 the second time, 144 the third time, and 432 the fourth or last time, all amounting to $640. Then say, as 640 : 320 :: 16 to $8, the first time. Again, as 640 : 320 :: 432 to $216, the last time.

Question 57.

As 10 : 1 :: 1 to $\frac{1}{10}$
As 20 : 1 :: 1 to $\frac{1}{20}$
As 40 : 1 :: 1 to $\frac{1}{40}$

Then $\frac{1}{10}+\frac{1}{20}+\frac{1}{40}=\frac{7}{40}$.
Then say, as $\frac{7}{40}$: 1 :: 1 to 5 min. $42\frac{6}{7}$ sec. Ans.

Question 58.

$\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{47}{60}$. Then say,

As $\frac{47}{60}$: 20 :: $\frac{20}{60}$ $(=\frac{1}{3})$ to $7\frac{1}{47}$ dolls.
As $\frac{47}{60}$: 20 :: $\frac{15}{60}$ $(=\frac{1}{4})$ to $5\frac{15}{47}$
As $\frac{47}{60}$: 20 :: $\frac{12}{60}$ $(=\frac{1}{5})$ to $4\frac{12}{47}$
As $\frac{47}{60}$: 20 :: $\frac{10}{60}$ $(=\frac{1}{6})$ to $3\frac{20}{47}$

Question 59.

Suppose the assembly consisted of 60 members; then half of 60 equal 30, and 20, (a third of 60,) add 10 equal 30, and 20+30=50. Then 60 less 50 equal 10, which 10 should equal the third of 60, and 10 absent: thus 30 less 10 equal 20, error too little.

Again, suppose 90, half of which is 45+20=65. Then 90÷3=30+10=40: then 90—65=25, should equal 40; therefore 40—25=15, error too little.

1st sup. 60 — 90 2nd sup.

Error 20 — 15 error. — Errors 20—15=5

1800
900

900

900 ÷ 5 = 180, the number. Ans.

Proof. 180÷2+20=110, and 180÷3+10=70; then 110 (met) + 70 (absent) = 180. Ans.

Question 60.

First vent in one hour will empty - - - - - 120 gallons.
Second vent in one hour will empty one-half 60
Third vent in one hour will empty one-third 40

Then three vents in one hour will empty 220 gallons. Then say,
As 220 : 1 :: 120 to 32 min. $43\frac{7}{11}$ sec. Ans.

Question 61.

6000 divided by 120 equal 50 cubic feet. 11325 oz. is the specific gravity of a cubic foot of lead: then 11325 multiply 50 equal 566250 oz. or 15 tons 15 cwt. 3 qrs. 26 lbs. 10 oz. Ans.

Question 62.

30 multiply 4,5 equal 135 feet, area of the side; and 135 multiply 2, (the centre of gravity,) equal 270: then 270 multiply 62,5, (the specific gravity in pounds of a cubic foot of running water,) equal 16875 lbs. or 7 tons 10 cwt. 2 qrs. 19 lbs. Ans. (See page 207.)

Question 63.

3 multiply 5 equal 15 square feet, the area of the bottom. 15 multiply 4 equal 60 cubic feet in the vessel: then 60 multiply 62,5, (the specific gravity in pounds,) equal 3750 lbs. or 1 ton 13 cwt. 1 qr. 26 lbs. Ans. (See page 207.)

Question 64.

100 yards per minute equal 5 feet per second. $5 \times 5 \div 64 = \frac{25}{64} = ,39$ of a foot for the perpendicular height, or fall of the water: then $62,5 \times ,39 =$ 24,375 lbs. the pressure upon each square foot: then 15 multiply 4 equal 60 feet, the area of the obstacle: then 24,375 multiply 60 equal 1462,5 lbs. going with the velocity of 5 feet per second: therefore 1462,5 multiply 5 equal 7312,5 lbs. Ans. (See page 207.)

Question 65.

1560,25 multiply 2 equal 3120,5; then 3161, (sum of their squares,) less 3120,5 equal 40,5 ÷ 2 equal 20,25, whose square root is 4,5, half the diff. and the square root of $\sqrt{1560,25}$ equal 39,5, (half sum,) add 4,5, (half diff.) equal 44, the greater number. Then 39,5, (half sum,) less 4,5, (half diff.) equal 35, the lesser number. Ans.

Question 66.

600 ÷ 8 equal 75; then 75 multiply 75 equal 5625 feet, the height the body must fall from, to acquire the velocity of 600 feet per second. Ans.

Rule for this and such like questions. The square root of the feet in the space fallen through, will always be equal to one-eighth of the velocity acquired at the end of the fall; therefore divide the velocity by 8, and the square of the quotient will be the distance fallen through, to acquire that velocity.

Question 67.

12 multiply 12 equal 144, the square of the semidiameter; then 144 multiply 2 multiply 20, and divide by 144, equal 40 feet, the solidity, when hewn square. Now, to find the whole solidity, 24 multiply 24, (whole diameter,) multiply ,7854 multiply 20 ÷ 144 = 62,8, the whole content, from which take 40, the content, when hewn square, and there remains 22,8 feet, the content of the slabs. Ans.

Rule. Multiply twice the square of the semidiameter by the length, and divide the product by 144, and the quotient will be the answer.

Question 68.

9 multiply 9 equal 81, and 5 multiply 10 equal 50 feet: then say,

As 5 : 81 :: 50 to 810, whose square root is 28,46 inches, the answer.

Question 69.

Weight of 2 passengers, 336 lbs. and of the body of the chaise, 84 lbs. making 420 lbs. 8 ft. × 12 = 96 inc. Then say,

As 96 : 420 :: 30 to $131\frac{1}{4}$ lbs. Ans.
As 96 : 420 :: 20 to $87\frac{1}{2}$ lbs.

Question 70.

A, B and C, can do it in 15 days. Then say,

As 15 : 1 :: 1 to $\frac{1}{15}$ by A, B and C.
As 30 : 1 :: 1 to $\frac{1}{30}$ by A.
As 40 : 1 :: 1 to $\frac{1}{40}$ by B.

Then $\frac{1}{30} + \frac{1}{40} = \frac{7}{120}$, and $\frac{1}{15} = \frac{8}{120}$; then $\frac{8}{120} - \frac{7}{120} = \frac{1}{120}$.
Then say, as $\frac{1}{120}$: 1 :: 1 to 120 days by C. Ans.

Question 71.

As 20 : 1 :: 1 to $\frac{1}{20}$ by A and B. As 12 : 1 :: 1 to $\frac{1}{12}$ by A+B+C. Then $\frac{1}{12}-\frac{1}{20}=\frac{1}{30}$. Then say,

As $\frac{1}{30}$: 1 :: 1 to 30 days by C. Ans.

Question 72.

$1,25 less 1,20 equal 5 cents loss on every 125 cents. Then say,

As 125 : 5 :: 100 to 4 dolls. Ans. 4 per cent.

Question 73.

As $115 : $160 :: $2,30 to $2, first cost per yard: then 2,40 less 2 equal 40 cts. profit on $2. Then say, as 2 : 40 :: 100 to 20 per cent. gain.

Question 74.

46 multiply 46 equal 2116, square of the greater, and 35 multiply 35 equal 1225, square of the lesser. 3341, sum of the squares. Ans.

1610 ÷ 46 = 35, the lesser number. 46, (the greater number,) less 35, (the lesser number,) equal 11, the difference: then 11 multiply 11 multiply 11 equal 1331, the cube of their difference. Ans.

Question 75.

$\frac{1}{3}+\frac{1}{6}=\frac{17}{18}$; then $1-\frac{17}{18}=\frac{1}{18}$. Then say,

As $\frac{1}{18}$: 12 :: $\frac{1}{1}$ to 216 feet, the answer.

Question 76.

$14,40 less $12,24 equal $2,16; then say, as 9 yds. : $2,16 :: 1 yard to 24 cents per yard, the answer.

Then $14,40 add $12,24 equal $26,64. Then say, as 24 cts. : 1 yd. :: $26,64 to 111 yards in both pieces. Ans.

Question 77.

81634×18×4=5834448 qts. then say, as $\frac{8}{9}$: 1 :: 5834448 to 6563754 bottles. Then 6563754 ÷ 144 = 45581 gross 7½ dozen. Ans.

Question 78.

As 6 : 10 :: 9 to 15 footmen: then 5 wagons at 25 cts. each is 125 cts. 9 horsemen at 12½ cts. each is 112½ cts. and 15 footmen at 6¼ cts. each, 93¾ cts. amounting in all to 331¼ cts. Then say, as 331¼ : 1 :: 1325 to 400. Then 400×5=2000 wagons, 400×9=3600 horsemen, 400×15=6000 footmen.

Question 79.

A 2, B 4, C 5, D 6, E 8: then 2+4+5+6+8=25, the sum. Then

As	25	:	2000	::	2 to $160 for A.
	25	:	2000	::	4 to 320 for B.
	25	:	2000	::	5 to 400 for C.
	25	:	2000	::	6 to 480 for D.
	25	:	2000	::	8 to 640 for E.

Question 80.

Suppose 20 admirals each $1, which is $20; the 30 post-captains each 80 cts. which is $24, the 24 sailing-masters each 50 cts. which is $12, and the 24 lieutenants each 33⅓ cts. which is $8. Then 20+24+12+8=64

Then say, as $64 : $256 :: $1 to $4.
64 : 256 :: 80 cts. to $3,20
64 : 256 :: 50 to 2
64 : 256 :: 33⅓ to 1,33⅓
} Paid each.

4×20=80 dolls. paid by 20 admirals. 3,20×30=96 dolls. paid by 30 post-captains. 2×24=48 dolls. paid by 24 sailing-masters. 1,33⅓×24=32 dolls. paid by 24 lieutenants. Ans.

Now to find their proportion. 4 admirals paid each 4 dolls. amounting to 16 dolls. 5 post-captains paid each 3 dolls. 20 cts. amounting to 16 dolls. 10 post-captains paid 32 dolls. 16 sailing-masters paid each 2 dolls. amounting to 32 dollars. 8 sailing-masters paid 16 dolls. and 12 lieutenants paid each 1 dollar 33⅓ cents, amounting to 16 dollars.

Question 81.

As 12 : 1 :: 1 to ,0833333 A + B + C
14 : 1 :: 1 to ,0714285 B + C + D
15 : 1 :: 1 to ,066666 A + C + D
18 : 1 :: 1 to ,055555 A + B + D

,276982

Then say, as ,276982 : 3 :: 1 to 10,8309505 days, all working together. Then 14—10,8309505=3,16905. Then say, as 3,16905 : 10,83095 :: 14 to 47,842 days by A. Then 15—10,83095=4,16905. Then say, as 4,16905 : 10,83095 :: 15 to 38,969 days by B. Then 18—10,83095=7,16905. Then say, as 7,16905 : 10,83095 :: 18 to 27,194 days by C. Then 12—10,83095=1,16905. Then say, as 1,16905 : 10,83095 :: 12 to 111,176 days by D.

Question 82.

Rule. From the sum take the difference, and half the remainder is the square of the less, which taken from the sum of their squares, will give the square of the greater.

208—80=128÷2=64; then 208—64=144. The square root of 144 is 12, the greater; and the square root of 64 is 8, the less number; and 12 less 8 equal 4, A had more than B. Ans.

Question 83.

Rule. From the square of their sum take the sum of their squares, and from the sum of their squares take this remainder, and the square root of the difference will be the difference of the two numbers.

50×50 equal 2500; then 2500 less 1300 equal 1200; then 1300 less 1200 equal 100, whose square root is 10, the difference of the two numbers.

50, (the sum,) ÷ 2 equal 25, half the sum; and 10÷2 equal 5, half the difference. Then to half the sum add half the difference, and from half the sum take half the difference, thus: 25 add 5 equal 30, the greater number; and 25 less 5 equal 20, the lesser number. Ans.

Question 84.

Suppose 1 boy got 1 dollar, 1 woman 2 dollars, and 1 man 4 dollars. Then 1×9 equal 9, 2×6 equal 12, and 4×4 equal 16, making 37. Then say,

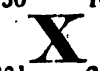
As 37 : 316,96⅔ :: 9 to $77,10÷9=$8,56⅔ cts. for each boy. Ans.
As 37 : 316,96⅔ :: 12 to 102,80÷8=17,13⅓ cts. for each woman.
As 37 : 316,96⅔ :: 16 to 137,63⅓÷4=34,96⅔ cts. for each man.

Question 85.

Suppose the greatest $150\times3=450$, and the lesser $116\frac{2}{3}\times5=583\frac{1}{3}$; then $583\frac{1}{3}-450=133\frac{1}{3}$, error.

Again, suppose the greatest $140\times3=420$, and the lesser $126\frac{2}{3}\times5=633\frac{1}{3}$; then $633\frac{1}{3}-420=213\frac{1}{3}$, error.

1st sup. 150 140 2nd sup.



Error $133\frac{1}{3}$ $213\frac{1}{3}$ error.

56000 96000
56000

$213\frac{1}{3}-133\frac{1}{3}=80\times3=240$, because the result of the multiplication is thirds.

$40000\div240=166\frac{2}{3}$, the greatest number; and $266\frac{2}{3}-166\frac{2}{3}=100$, the lesser number. Ans.

PROOF. $100\times5=500$, and $166\frac{2}{3}\times3=500$.

Question 86.

Suppose his age 30 years. She must be 15 years.
Lived together 30 30

He must be 60 years. She must be 45 years.

Then his age was to her's, as 2 to $1\frac{3}{7}$. Then say, as $1\frac{3}{7}:2::45$ to 63 years he should be. Then 63 less 60 equal 3, error.

Again, suppose he was 20 yrs. She must be 10 yrs.
Lived together 30 30
-50 -40

Then say, as $1\frac{3}{7}:2::40$ to 56 yrs. he should be. Then $56-50=6$, error.

1st sup. 30. 20 2nd sup.

Error 3 6 error. Errors, 6 less 3 equal 3.

60 180
60

$-120\div3=40$ yrs. he was, when married, and consequently she must be 20 years. Ans.

PROOF. When married, he was double her age, that is, as 1 to 2; therefore $40\div2=20$, her age. Then 30 years after, his was to her's as 2 to $1\frac{3}{7}$. Then he was 70, and she was 50; thus, as $2:1\frac{3}{7}::70$ to 50, her age at that time. Then 30 years more makes him 100, and her 80; and his proportion to her is as 2 to $1\frac{3}{4}$; thus, as $2:1\frac{3}{4}::100$ to 80, her age at that time.

Question 87.

RULE. The velocity of the two hands of a watch or clock, are to each other as 12 to 1; therefore the difference of velocity is $12-1=11$. Then say,

As 11 : 1 :: 12 × 1 to 1*h.* 5*m.* $27\frac{3}{11}$ *sec.* Ans.
11 : 1 :: 12 × 2 to 2 10 $54\frac{6}{11}$

Question 88.

RULE. From the sum of their squares take the square of their difference, and to the sum of their squares add the remainder, and the square root of this sum will be the sum of the required numbers. Then to half the sum add half the difference, and from half the sum take half the difference, and your will have the two numbers separately

12500—50×50=10000+12500=22500, whose square root is 150, sum of the two numbers: then 150÷2 equal 75, the half sum: then 50 is their difference: then 50÷2 equal 25, half difference: then to 75, half the sum, add 25, half the difference, making 100, the number belonging to A; and 75 less 25 equal 50, the number belonging to B. Ans.

Question 89.

14 add 14 equal 28, what both had; then 336÷28 equal 12, the difference. Then 28 ÷ 2 equal 14, half sum; and 12 ÷ 2 equal 6, half the difference. Then 14 add 6 equal 20, the number Tom retired with: then 14 less 6 equal 8 Jack had still; therefore it will be that Jack lost 14—8=6 he lost. Ans.

Question 90.

288÷12 equal 24, the sum, and 24÷2 equal 12, the half sum, and 12, the difference: then to half the sum add half the difference, and from half the sum take half the difference, to find the two numbers. 12 is half the sum, and 12 ÷ 2 equal 6, half the difference: then 12 add 6 equal 18, William's share, and 12 less 6 equal 6, John's share. Ans.

Question 91.

RULE for all such questions. As unity is to the distance, so is the number in rank, less by one, to a fourth number. Next do the same with the file, and multiply the two numbers found together by the proportion stated, and the product will be the answer, thus:

As 1 : 4 :: 56—1 to 220. As 1 : 4 :: 36—1 to 140.

Then 220 × 140 equal 30800 square feet. Ans.

Question 92.

RULE for all such questions. Say as the ratio in length is to the ratio in breadth, so is the number of trees to a fourth number, whose square root is the number in breadth; and as the ratio in breadth is to the ratio in length, so is the number of trees to a fourth number, whose square root is the number in length, thus:

As 3 : 2 :: 600 to 400, whose square root is 20.
As 2 : 3 :: 600 to 900, whose square root is 30.

That is, 20 in breadth, and 30 in length. Ans.

Then say, as 1 : 7 :: 20—1 to 133. As 1 : 7 :: 30—1 to 203.

Then 203 multiply 133, equal 26999 square yards. Ans.

Question 93.

6 multiply 6 equal 36, and 4 multiply 4 equal 16, multiply 3 equal 48: then say inversely, as 36 : 4 :: 48 to 3 hours. Then for double say, as 1 : 3 :: 2, (direct proportion,) to 6 hours. Ans.

Question 94.

60 multiply 60 equal 3600, and 37 multiply 37 equal 1369, and 23 multiply 23 equal 529: then 3600 less 1369 equal 2231, square root thereof, 47,23+. Then 3600 less 529 equal 3071, whose square root is 55,41+. Then 47,23 add 55,41 equal 102,64 feet. Ans.

Question 95.

4 add $1\frac{1}{2}$ equal $5\frac{1}{2}$ miles per hour, down the river; and 4 less $1\frac{1}{2}$ equal $2\frac{1}{2}$ miles per hour, up the river. Then $5\frac{1}{2}$ and $2\frac{1}{2}$ equal 8 miles per hour, by the two boats. Then say, as 8 : 1 :: 20 to $2\frac{1}{2}$ hours. Then as the two boats will complete the 20 miles in $2\frac{1}{2}$ hrs. and the boat from Wheeling goes at the rate of $5\frac{1}{2}$ miles an hour, and the boat going to Wheeling, goes at the rate of $2\frac{1}{2}$ miles an hour, it will be thus: $5\frac{1}{2}\times 2\frac{1}{2}=13\frac{3}{4}$ miles by the boat from Wheeling, and $2\frac{1}{2}\times 2\frac{1}{2}=6\frac{1}{4}$ miles by the boat going to Wheeling. Ans.

PROOF. $13\frac{3}{4}$ add $6\frac{1}{4}$ equal 20 miles, the distance.

Question 96.

1728 inches in a solid foot; then $1728\times 4=6912$ inc. $\div 12=576$ feet.

$,25\times,25\times 4=\frac{1}{4}$, or ,2500. $\frac{1}{4}$ inch square will take 4 inches in length to make a cubic inch.

Question 97.

Her age $4\times 4=16$; then say, as $\frac{4}{7}$: 16 :: 1 to 36: then $36\div 3=12$: then say, as $\frac{2}{7}$: 12 :: 1 to 42: then $42\div 3=14$ years. Ans.

PROOF. Her age 14; then $14\times 3=42$, and $\frac{2}{7}$ of $42=12$, and $12\times 3=36$, and $\frac{1}{4}$ of $36=16$, whose square root is 4.

Question 98.

RULE. To half the sum add half the difference, and that sum is the greatest number; and from half the sum take half the difference, and the remainder is the least number.

48 (sum) $\div 2=24$, half sum; then 14 (diff.) $\div 2=7$, half the difference. Then $24+7=31$, greater number; and $24-7=17$, less number. Ans.

Question 99.

This question is to measure a quadrant, or the fourth part of the circumference of a circle. The height of the signpost is the radius, or semidiameter of the circle; therefore it will be 40 feet, equal to the semidiameter; and $40\times 2=80$, the whole diameter. Now, to find the circumference, say,

As 7 : 22 :: 80 to $251\frac{3}{7}$ feet, the cir. Then $251\frac{3}{7}\div 4$, (the fourth part of the cir.) equal $62\frac{5}{7}$ feet, the curve line formed by the top-end of the signpost, in falling. Now, as 1 : 1 :: $62\frac{5}{7}$ to 62 days 20 hrs. $34\frac{3}{7}$ min. Ans.

Question 100.

$A+B\ \frac{3}{11}=,2727$
$A+C\ \frac{4}{13}=,3846$
$B+C\ \frac{2}{7}=,2857$
,9430

$A+B+C=,4715$
$A+B=,2727$
$C=,1988$

$A+B+C=,4715$
$A+C=,3846$
$B=,869$

Then ,9430 $\div 2$, (the number of combinations,) $=,4715$.

$A+B+C=,4715$
$B+C=,2857$
$A=,1858$

Then say, as 4715 : 10 :: 1988 to $4 21$\frac{2980}{4715}$ cts. for A.
4715 : 10 :: 1858 to 3 94$\frac{2790}{4715}$ cts. for B.
4715 : 10 :: 869 to 1 84$\frac{1440}{4715}$ cts. for C.

Proof $10 00

Question 101.

Suppose the distance 10 miles, from Washington to Cambridge; then Endley's horse will be 10 hours in hauling the load of salt, that is, a mile an hour. Endley's and Laurence's will haul the same load in 6 hours, that is, a mile and two-thirds per hour by the two horses. Endley's and Skinner's will haul the same in 4 hours, that is, $2\frac{1}{2}$ miles per hour by them two. Now, to find the rate per hour for Laurence's horse, and Skinner's.

Endley's in 10 hours, that is, $10 \div 10 = 1$ mile per hour. E's and L's in 6 hours, that is, $10 \div 6 = 1$ and two-thirds per hour. E's and S's in 4 hrs. that is, $10 \div 4 = 2\frac{1}{2}$ per hour. Now Endley's and Laurence's went a mile and two-thirds an hour, and Endley's went alone a mile an hour; therefore Laurence's horse will go two-thirds of a mile an hour, and E's and S's travel $2\frac{1}{2}$ miles an hour. E's will go or travel alone one mile an hour; therefore S's horse will travel $1\frac{1}{2}$ miles an hour. Then the proportions are—Endley's one mile, Laurence's two-thirds of a mile, Skinner's a mile and a half per hour.

Then $1 + \frac{2}{3} + 1\frac{1}{2} = 3\frac{1}{6}$. Then say,

					$	cts.	m.		
As $3\frac{1}{6}$	:	200	::	$1\frac{1}{2}$ to	94	78	$6\frac{16}{19}$	W. S.	Ans.
As $3\frac{1}{6}$	:	200	::	1 to	63	15	$7\frac{17}{19}$	J. E.	
As $3\frac{1}{6}$	:	200	::	$\frac{2}{3}$ to	42	10	$5\frac{5}{19}$	J. L.	

Question 102.

Rule. From the centre to the given height being $1\frac{1}{2}$ semidiameter, multiply the square of $1\frac{1}{2}$ by the weight, and the product will be the answer.

$1{,}5 \times 1{,}5 \times 400 = 900$ lbs. Ans.

Question 103.

$180 \div 20 = 9$, whose square root is 3, that is, 3 semidiameters from the earth's centre, which is equal to $4000 \times 3 = 12000$ miles from the earth's centre. Then 12000 less 4000, (the earth's semidiameter,) equal 8000 miles above the surface of the earth. Ans.

Question 104.

The proportion is as 1 is to 2, to lose half its weight; therefore it will be, As 1 : 4000×4000 :: 2 to 32000000. Then $\sqrt{32000000} = 5656{,}85$ miles from the centre of the earth. Then 5656,85 less 4000, (the earth's semidiameter,) equal 1656,85 miles above the surface, or in the air. Ans.

Question 105.

As 13 : 1 :: 1 to $\frac{1}{13}$. As 8 : 1 :: 1 to $\frac{1}{8}$.

Then $\frac{1}{8} - \frac{1}{13} = \frac{5}{104}$; then as $\frac{5}{104}$: 1 :: 1 to $20\frac{4}{5}$ days by B.

Question 106.

1 mile equals 320 rods; then 320 multiply $320 \div 160 = 640$ acres; and 320 multiply 4 multiply 10, equal 12800 rails: then 12800 multiply $12800 \div 640$, equal 256000 rails, which will enclose 256000 acres, which is 20 miles square.

Question 107.

Pepper 6 lbs. = 13 lbs. ginger. Ginger 19 lbs. = 4¼ lbs. cloves. Cloves 10 lbs. = 63 lbs. sugar, at 5 cts. a pound. Then $13 \times 4{,}75 \times 63 \times 5 \times 100 = 1945125$. Then 6 multiply 19 multiply 10, equal 1140. Then $1945125 \div 1140 = 1706{,}25$ cts. which is 17 dolls. 6¼ cts. per 100 lbs. Ans.

Question 108.

As 13 is added to the number, it will be $45 + 13 = 58$: then say, as 45 : 80 :: 58 to 103 dollars $11\frac{8}{45}$ cents in the same time; and in twice that time, the 58 persons will spend 103 dolls. $11\frac{1}{9}$ cts. $(\frac{8}{45} = \frac{1}{9},) \times 2 = 206$ dolls. $22\frac{2}{9}$ cts. Then say, as $\$206{,}22\frac{2}{9}$ cts. : 70 :: $\$252$ to 85 dolls. 53 cts. 5+ mills per hhd. which $\times 2 = 171$ dolls. 7 cts. nearly. Ans.

Question 109.

$22 \times 4 = 88$ miles, the clerk is before the messenger; and $32 - 22 = 10$ miles, the messenger gains on the clerk each day. Then say, as 10 : 1 :: 88 to $8\frac{8}{5}$ days. Then $8\frac{8}{5} \times 32 = 281\frac{3}{5}$ miles, the messenger travelled, before he overtook the clerk. The whole distance $400 - 281\frac{3}{5} = 118\frac{2}{5}$ miles this side of Philadelphia. Ans.

Question 110.

Three-eighths of 2 dolls. 20 cts. equal $82\frac{1}{2}$ cts. gained per yard, when sold at 2 dolls. 20 cts. Then 2 dolls. 20 cts. less $82\frac{1}{2}$ cts. equal $137\frac{1}{2}$ cts. per yard, first cost. Raised to 2 dolls. 70 cts. from which take 1 dol. $37\frac{1}{2}$ cts. first cost, and the gain is 1 dol. $32\frac{1}{2}$ cts. Then say, as $137\frac{1}{2} : 132\frac{1}{2} :: 100$ to 96 dolls. 36 cts. 3+ mills per cent. Ans.

Question 111.

The son's share to the mother's, is as 2 to 1, and the mother's to the daughter's, is as 2 to 1; therefore their 3 shares are 4, 2, 1, which added is 7. Then of course their real shares are $\frac{4}{7}$, $\frac{2}{7}$, and $\frac{1}{7}$. Now $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$; therefore say,

As $\frac{8}{21}$: 8000 :: $\frac{1}{3}$ to 7000 dolls. Ans.

Question 112.

Suppose A's gain to be 2; then, by the question, B's is 3, and C's will be 4, making 9, the sum. Then say,

As 9 : 936 :: 2 to 208 A's gain. Ans.
9 : 936 :: 3 to 312 B's
9 : 936 :: 4 to 416 C's

$3 \times 5 \times 7 = 105$, common denominator. Then say,

As 3 : 2 :: 1 to $\frac{2}{3}$; then $\frac{2}{3}$ of 105 = 70 A's
5 : 3 :: 1 to $\frac{3}{5}$; then $\frac{3}{5}$ of 105 = 63 B's
7 : 4 :: 1 to $\frac{4}{7}$; then $\frac{4}{7}$ of 105 = 60 C's

193 sum.

Then say, as 193 : 15440 :: 70 to $5600 A's stock.
193 : 15440 :: 63 to 5040 B's stock.
193 : 15440 :: 60 to 4800 C's stock.

Question 113.

Suppose his age, when married, 27 years. Her's must be 9 yrs.

	15	15
	42	24
Then his should double her's,	48	2
Error too little by	6 yrs.	Double of her's 48

Again, suppose his age 30 years. Her's must be 10 yrs.

	15	15
	45	25
	50	2
Error too little by	5 years.	50

1st sup. 27 30 2nd sup.

Error 6 5 error. Errors, 6—5=1, divisor.

From 180 135
Take 135

45 ÷ 1 = 45 years, his age, when married.

Now find her age by this proportion: As 3 : 2 :: 45 to 15 years, her age, when married. Then 45 + 15 = 60, his age, when married 15 years. Then find her age by the following proportion:

As 2 : 1 :: 60 to 30 years, her age, when married 15 years.

Therefore, when married, he was 45, and his wife 15. Ans.

Question 114.

As 2 : 5 :: 2000 to 5000 for B; then say,

As 4 : 7 :: 5000 to 8750 for A.

Then A got $8750, B 5000, and C 2000, making the whole legacy 15750 dollars. Ans.

Question 115.

3×3=9, the square of A's distance; and 6×6=36, the square of B's distance. Then,

As 9 : 1 :: 36 to 4. A is 4 times as warm as B. Ans.

Question 116.

The velocity of the ram is 24 feet, and the weight of the ball 42 lbs. Compounded make a fraction of $\frac{24}{42}=\frac{4}{7}$.

Then it will be $\frac{4}{7}\times 8000=4571\frac{4}{7}$ feet in a second. Ans.

Question 117.

As 1 : 90 :: 126 to 113 dollars 40 cts. Then the first cost was $132 — 113,40 = 18 dollars 60 cents. Then say,

As 90 : 1 :: 18,60 to $20\frac{2}{3}$ gallons of water. Ans.

Question 118.

1st. Suppose the body was 40 inc. long, then the tail must be 20 + 8 = 28.

Length of tail 28 inches.
Head 8

36 40 — 36 = 4, error.

2nd. Suppose the body was 60 inc. long, then the tail must be 30 + 8 = 38.

Length of tail 38 inches.
Head 8

46 60 — 46 = 14, error.

1st sup. 40 60 2nd. sup.

Error 4 14 error. Errors, 14 — 4 = 10, divisor.

240 560
240

320 ÷ 10 = 32 inches, the length of the body.

The length of the body was 32 inc.
The length of the head was 8
Tail equals half the body and head, that is, 32 ÷ 2 + 8 = 24

64 inc. whole length of the fish. Ans.

Proof. The body was 32 inches long, just as long as the head and tail. The head 8 inc. and tail 24 inc. making 32 inc.

Question 119.

They gain 1000 dolls. A's gain 550 dolls.
B's share 450 B's gain 450

A's gain 550 100 diff.

As 100 : 192 :: 550 to 1056 A's ÷ 8 = 132 dolls.
As 100 : 192 :: 450 to 864 B's ÷ 8 = 108

A's was valued at $132, and B's at $108 per hhd. Ans.

Question 120.

180 ÷ 4 = 45, and 100 — 6 = 94; then 94 : 100 :: 50 to 53 dolls. $19\frac{1}{94}$ cts. Then $53,19\frac{1}{94} - 45 = 8$ dolls. $19\frac{1}{94}$ cts. Then say, as $53,19\frac{1}{94} : 8,19\frac{1}{94}$ :: 100 to 15 dollars 39 + cents per cent. loss. Ans.

I omitted bringing in the fractions in the 1st and 2nd numbers.

Question 121.

As $3\frac{1}{2}$: 100 :: 154 to $4400 for one year. Then say inversely, as 1 : 4400 :: 1,25 to 3520 dolls. Ans.

Question 122.

The barrel contained 32 gallons, from which take 5, and there will remain 27. Then say,

As 32 : 27 :: 5 to $4\frac{7}{32}$ gallons of whiskey.

Then $27 - 4\frac{7}{32}$ (gallons taken out) $= 22\frac{25}{32}$ gallons whiskey in the barrel yet. Then say,

As 32 : $22\frac{25}{32}$ :: 5 to $3\frac{573}{1024}$ gallons whiskey.

Then $22\frac{25}{32} - 3\frac{573}{1024} = 19\frac{327}{1024}$ gallons of whiskey in the barrel yet. Then say,

As 32 : $19\frac{327}{1024}$ :: 5 to $3\frac{111}{32768}$ galls. of whiskey taken out. Then $19\frac{327}{1024} - 3\frac{111}{32768} = 16\frac{7153}{32768}$ gallons of whiskey in the barrel yet. Then say,

As 32 : $16\frac{7153}{32768}$ :: 5 to $2\frac{560053}{1048576}$ gallons whiskey taken out.

Taken out the first time, 5 gallons of whiskey.

The second time, $4\frac{7}{32}$

The third time, $3\frac{573}{1024}$

The fourth time, $3\frac{111}{32768}$

The fifth time, $2\frac{560053}{1048576}$

Total $18\frac{331157}{1048576}$ gallons taken out.

From 32 gallons, contents of the barrel,
Take $18\frac{331157}{1048576}$ gallons taken out.

Ans. $13\frac{717419}{1048576}$ gallons remaining.

Of course there must be as much water in, as will make the difference to 32 gallons; therefore it will be,

$32 - 13\frac{717419}{1048576} = 18\frac{331157}{1048576}$ gallons of water, equal to the quantity of whiskey taken out. Ans.

Question 123.

$4 \times 4 \times 4 \times 4 = 256$, and $5 \times 5 \times 5 \times 5 \times 5 = 3125$.

Then $3125 \div 256 = 12\frac{53}{256}$, the number. Ans.

Question 124.

This question is, the sum of any two numbers, and their products being given, to find each number.

RULE. From the square of their sum take four times their product, and extract the square root of the remainder, which will be the difference of the two numbers: then half the difference added to half the sum, gives the greater of the two numbers; and half the difference taken from half the sum, gives the lesser number.

604×604=364816 sum squared.
2400×4= 9600 four times the product.

$\sqrt{355216}$=596, the difference.

Then the sum 604÷2=302, half sum. The difference 596÷2=298, half difference. Then 302+298=600, the greater number, or number of yards bought.

To find the lesser number, it will be 302—298=4, the lesser number, or price per yard.

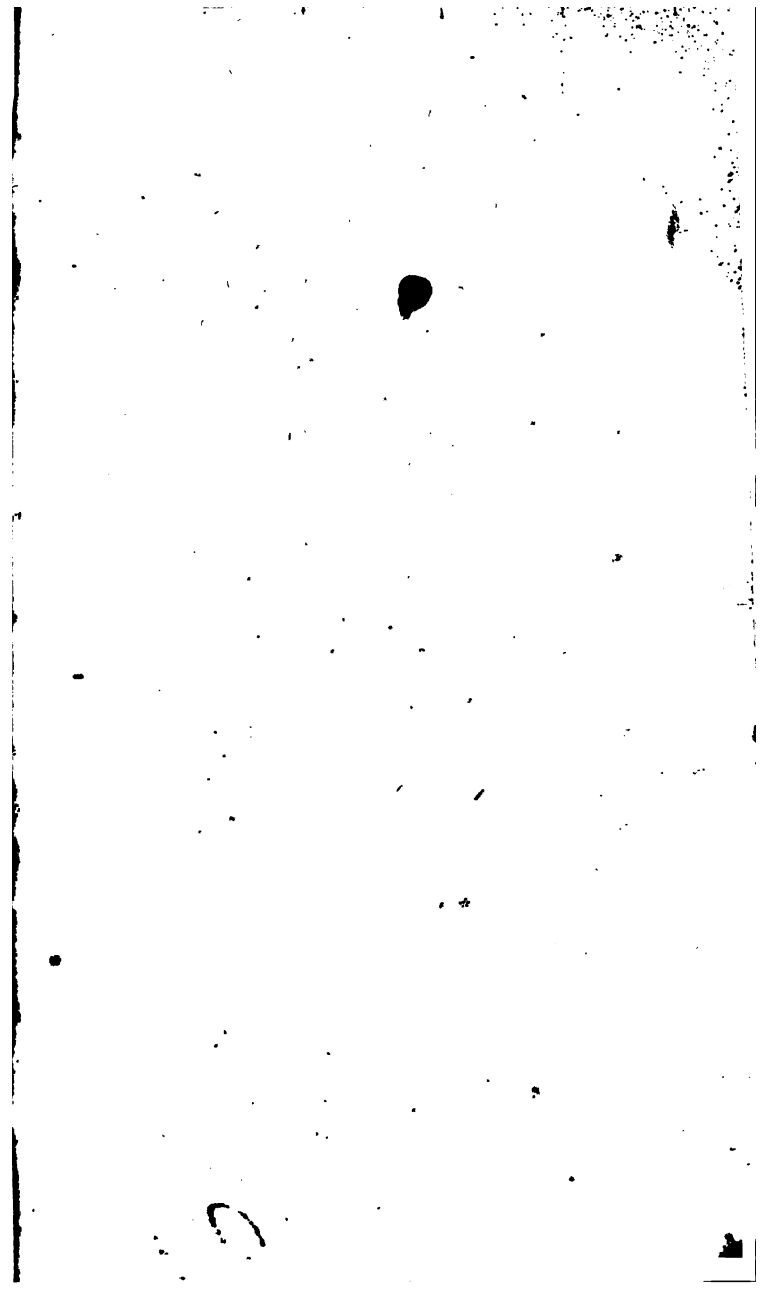
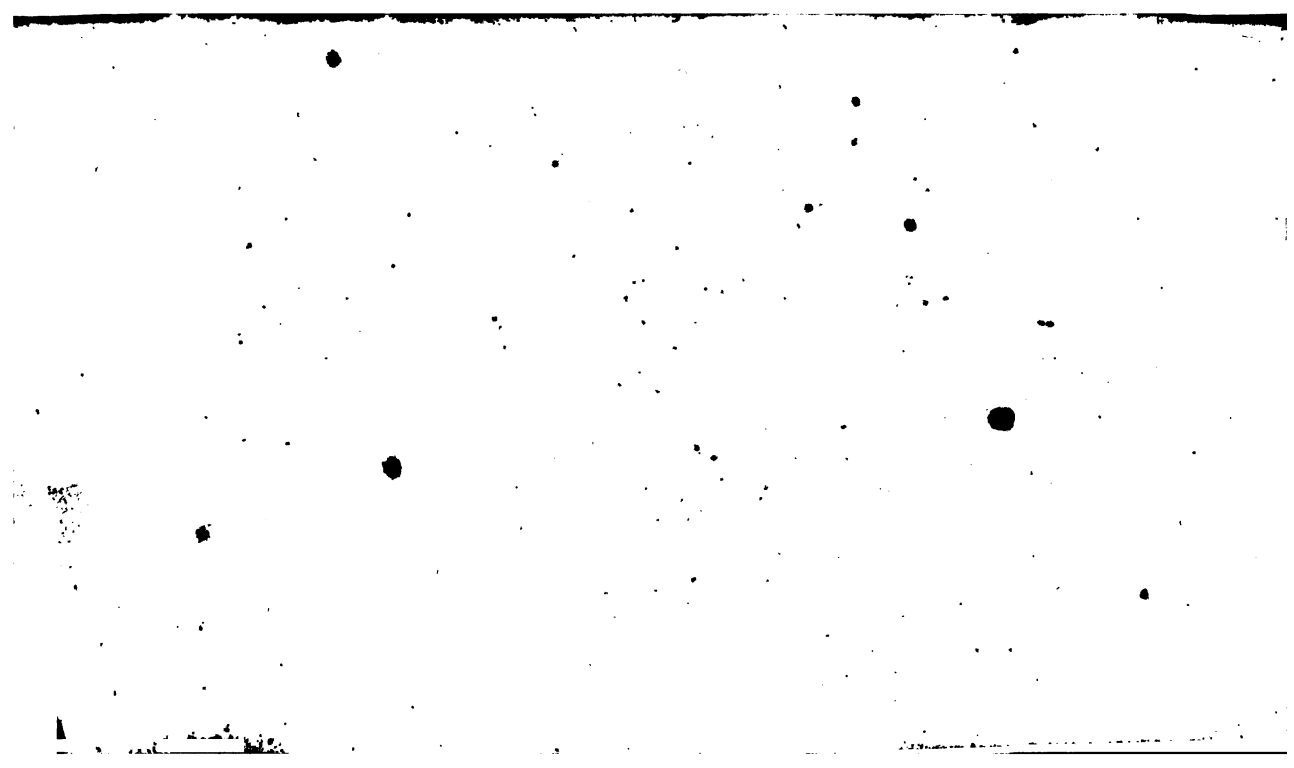
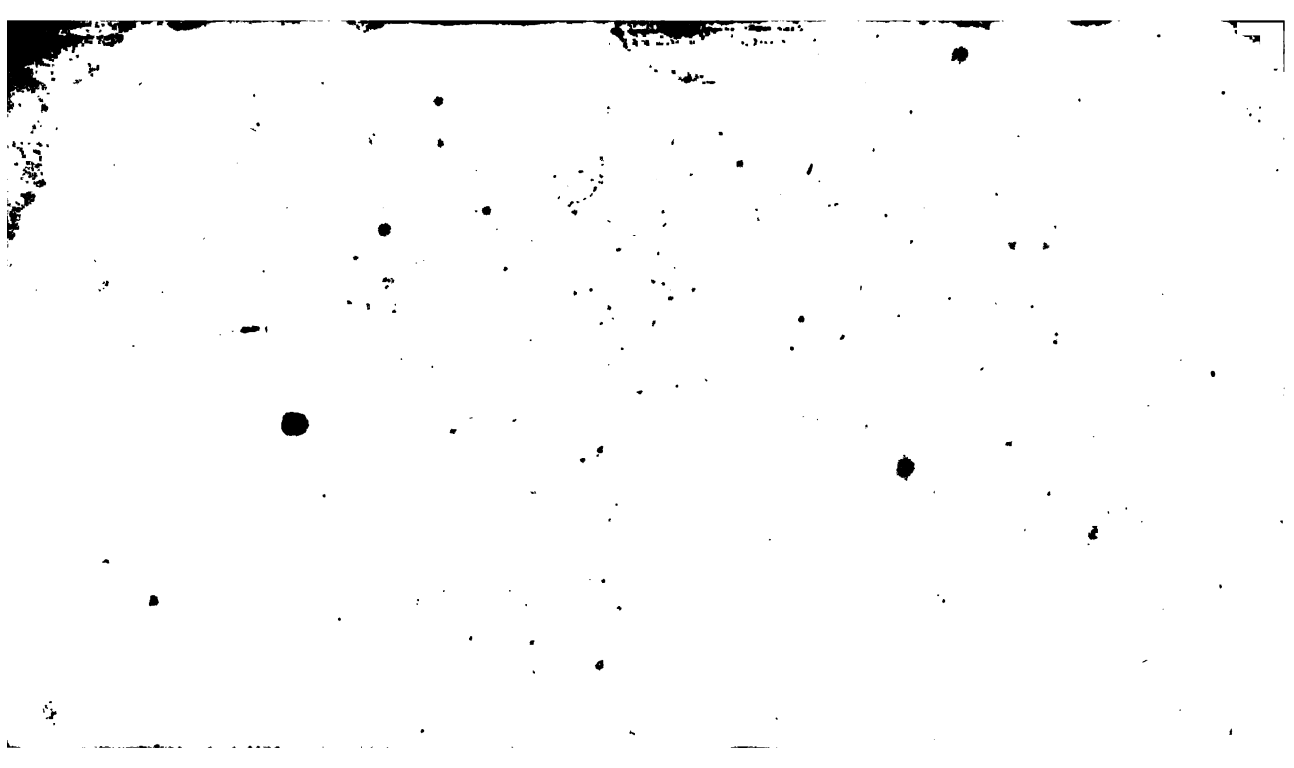
Ans. 600 yards, at 4 dollars per yard.

Water is a liquid that in Egypt is found;
Sloe is a fruit that grows not far from the ground;
Hat is an article that is useful to man;
Ink is a fluid that is us'd on the ocean;
Grape is a fruit that in Asia doth grow;
Thames is a river that in Europe doth flow;
Ostrich, a bird that is rare to be found;
Nut, a fruit that grows on American ground.

Now begin at the first line, and continue to the last, and take the given fractional parts of the first word in each line, and arrange them towards the right hand, thus:

Two-fifths of *water* is - - - - - WA
One-fourth of *sloe* is - - - - - - S
One-third of *hat* is - - - - - - - H
Two-thirds of *ink* is - - - - - - IN
One-fifth of *grape* is - - - - - - G
One-sixth of *Thames* is - - - - - - T
One-seventh of *ostrich* is - - - - - - O
One-third of *nut* is - - - - - - - - N.

Ans. WASHINGTON.

NOTE. This was the first question of the sort ever published in this country. After I shewed several how to solve such, and form a rebus, there afterwards appeared two simple ones in the Washington Republican, a paper printed in this town. The learner will observe, that a rebus and an enigma are not the same: the rebus is solved as above, whereas the enigma, or riddle, is only guess-work.

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